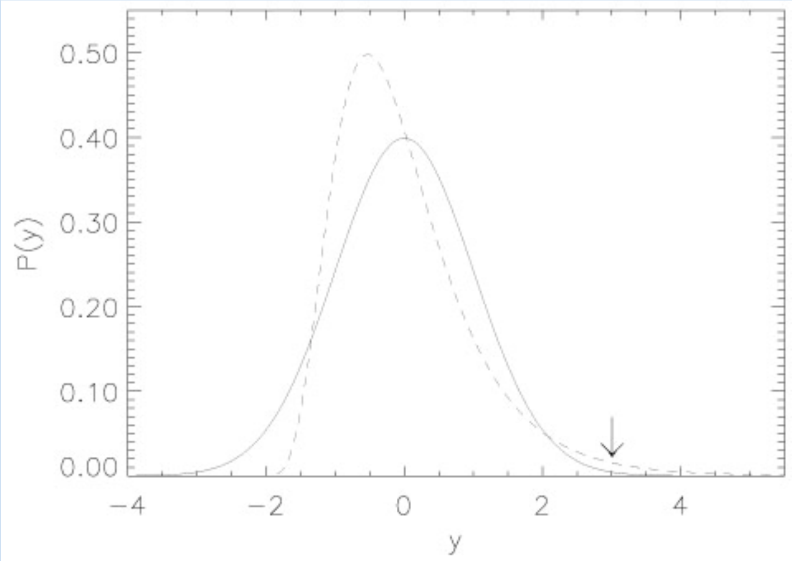
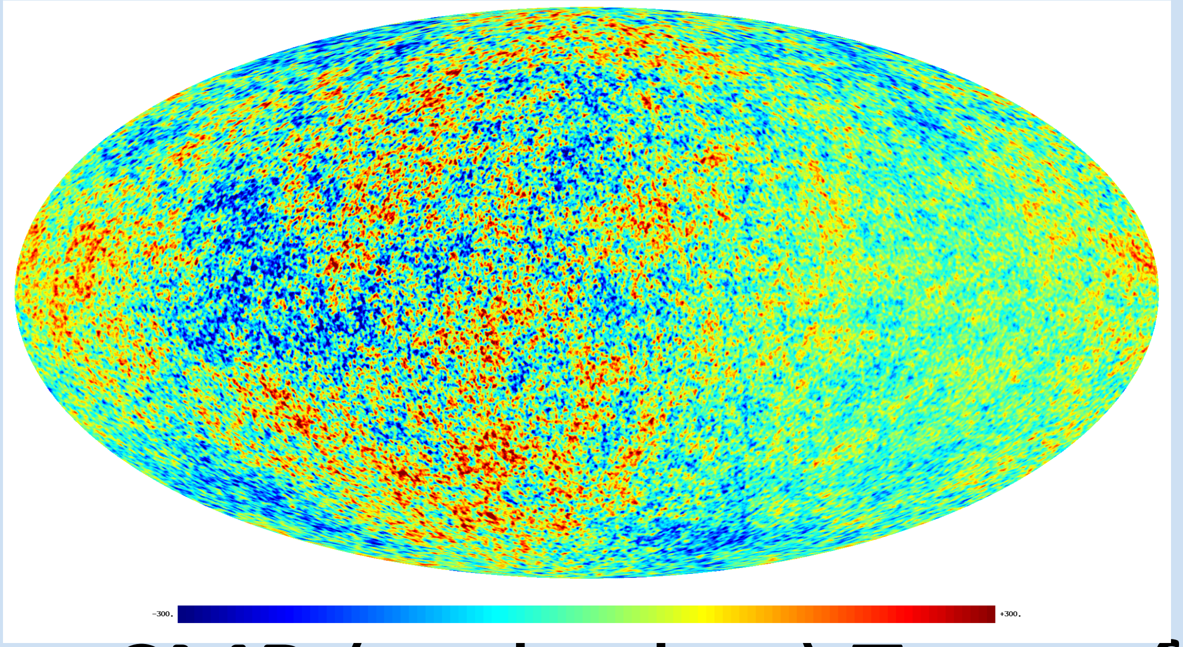


Beyond Gaussian Fluctuations in Galaxies and the CMB

Marc Kamionkowski
(Caltech)

LBL, 10 March 2011



New CMB (and other) Tests of Inflation, Dark Energy, and other novel physics

Marc Kamionkowski

(Caltech)

Berkeley, 20 September 2010

Cosmological birefringence
Statistical isotropy
Power inhomogeneities

This talk:

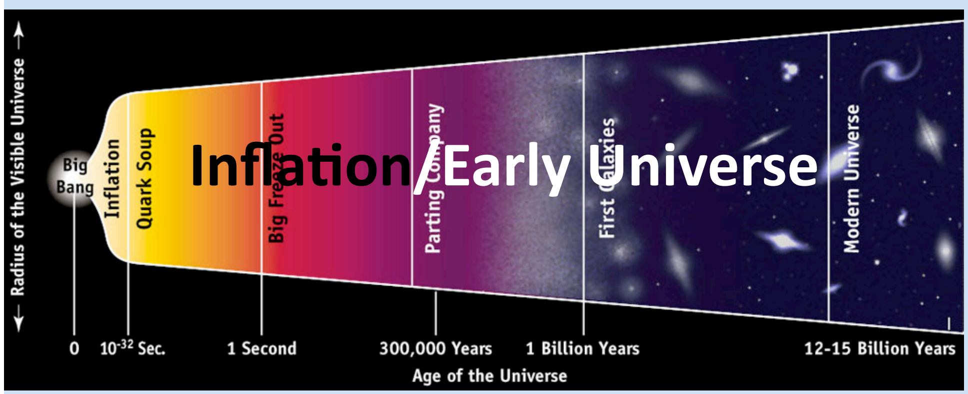
Non-gaussianity from self-ordering scalar fields (w. Caldwell, Figueroa, arXiv:1003.0672)

Scale-dependent halo bias (w. Schmidt, 1008.0638)

Odd-parity bispectra (w. Souradeep, 1010.4304)

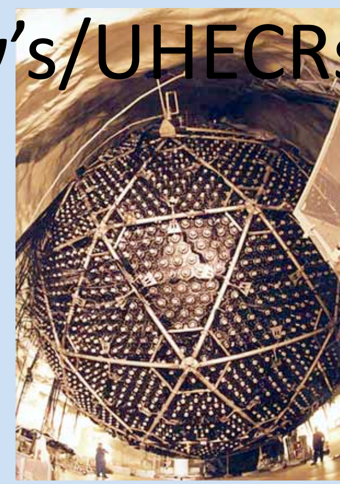
Statistics of CMB non-Gaussianity (w. Smith, Heavens, 1010.0251 and w. Smith, in preparation)

Particle-Astro Interface:

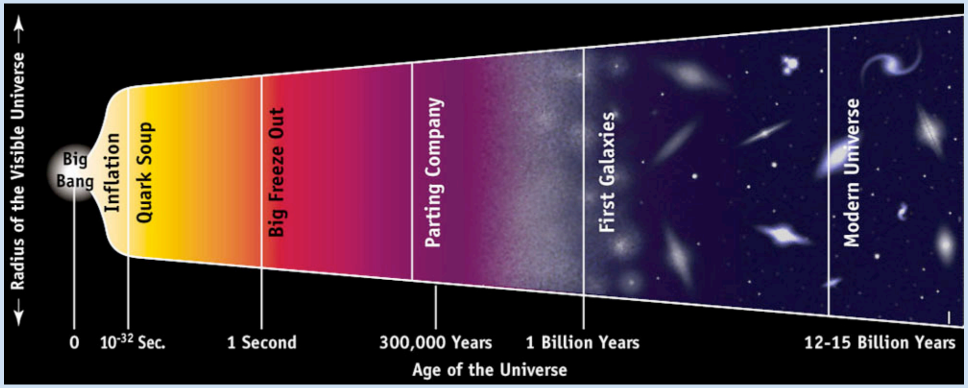


Inflation/Early Universe

ν's/UHECRs



This talk:



Inflation/Early Universe

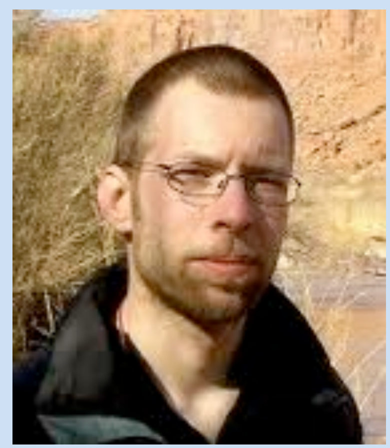
Who did the work

Tristan
Smith

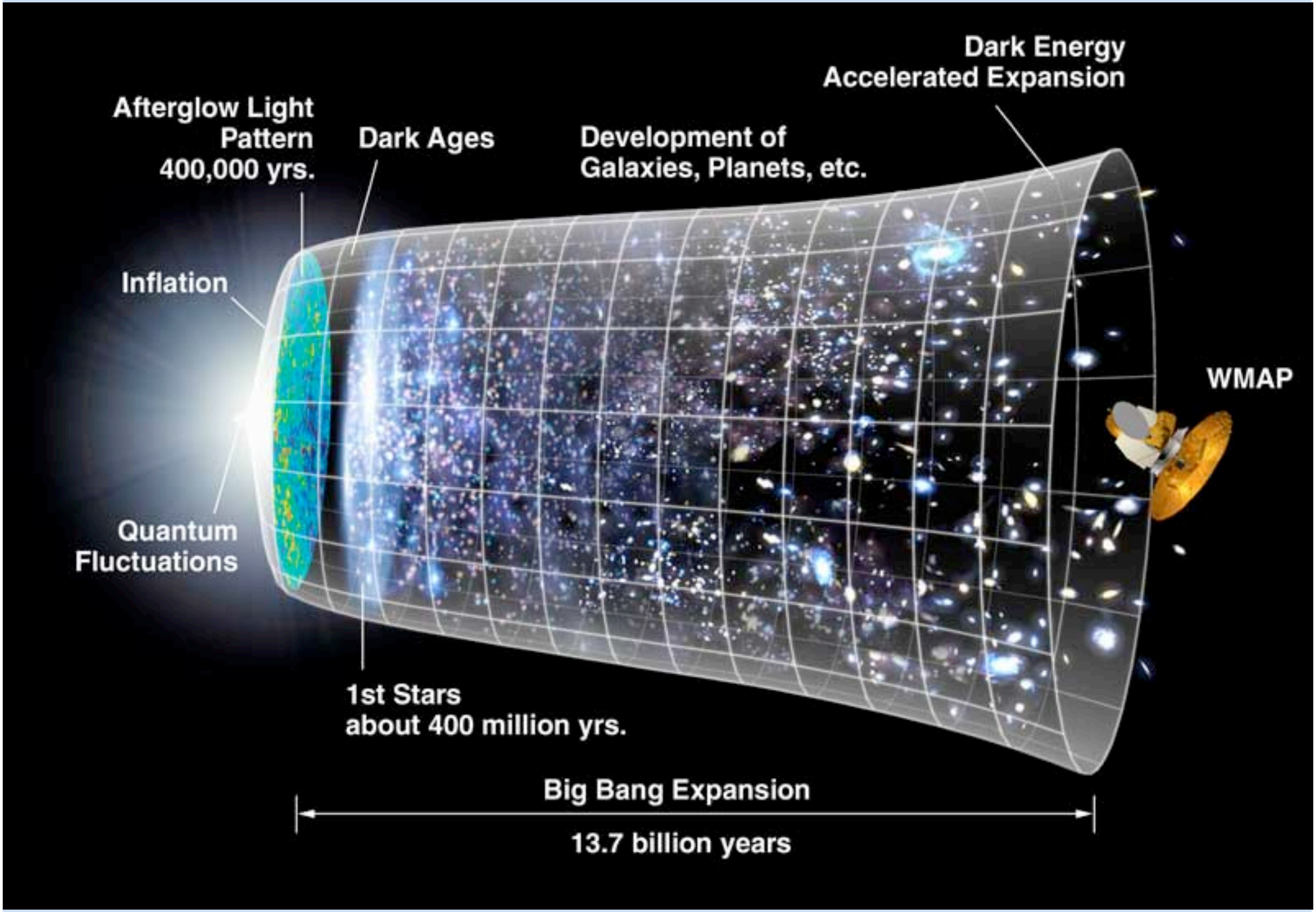
Robert Caldwell

Tarun Souradeep

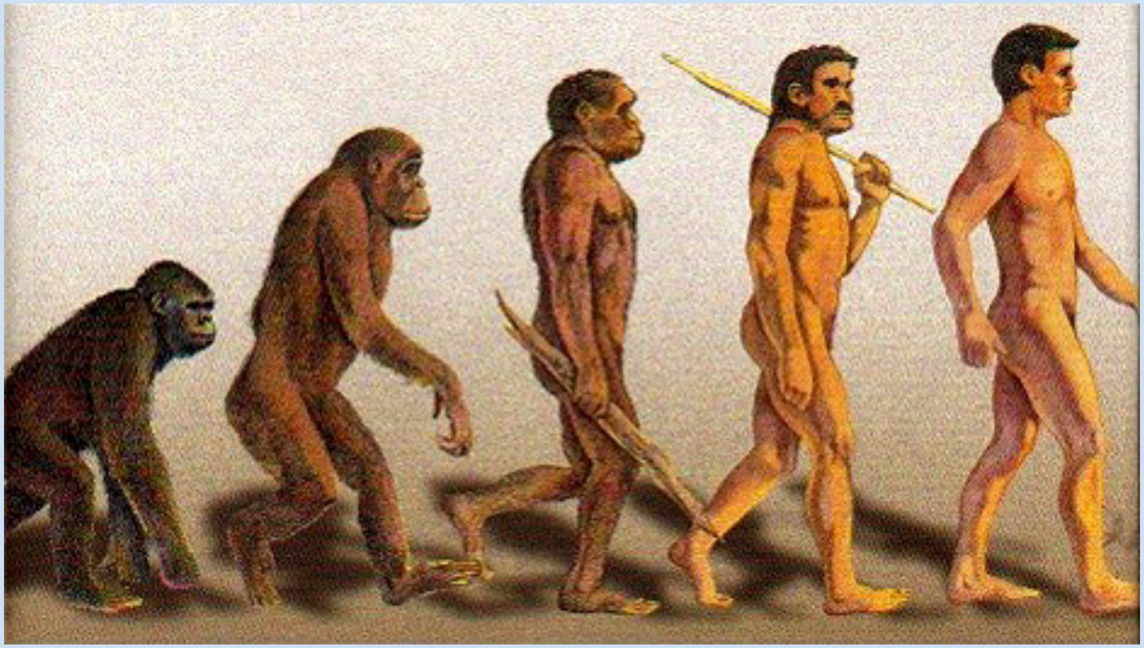
Dani Figueroa

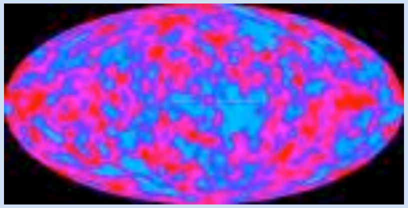


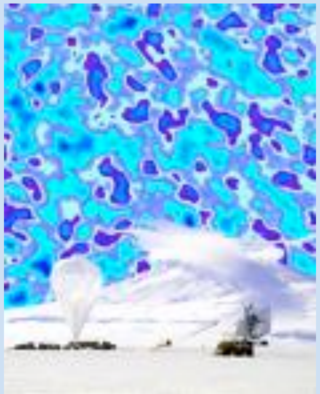
Fabian Schmidt

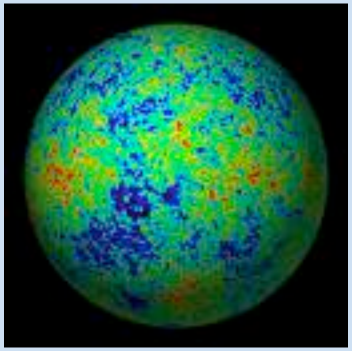


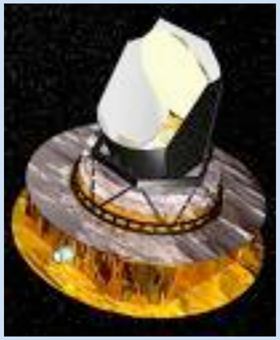
Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years











AT&T:
1965

COBE:
1991

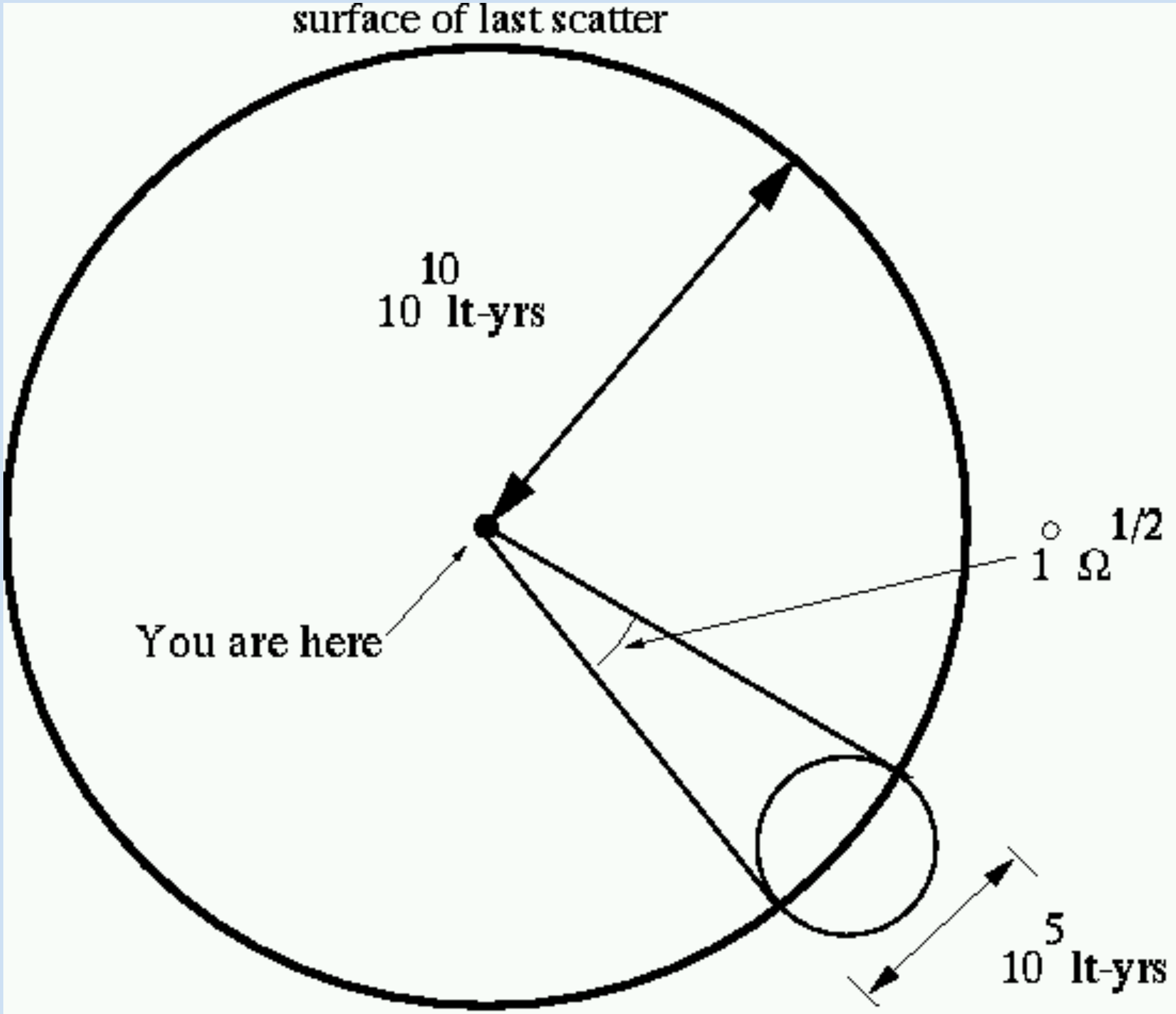
BOOMERanG:
2000

WMAP:
(2003-present)

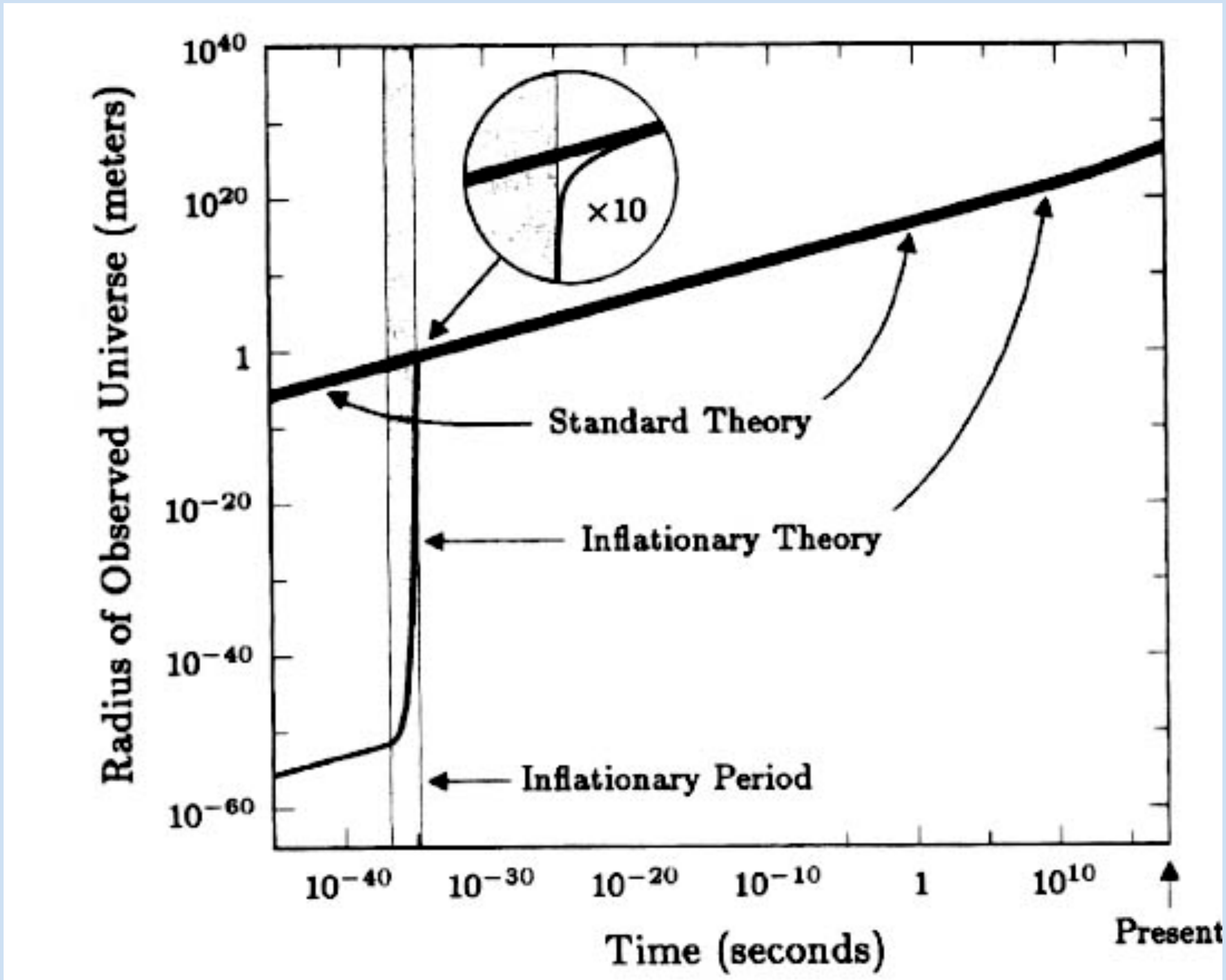
Planck:
Even better!!
(launched
May 2009)

…and beyond!!

Isotropy problem: Why is the Universe so smooth?

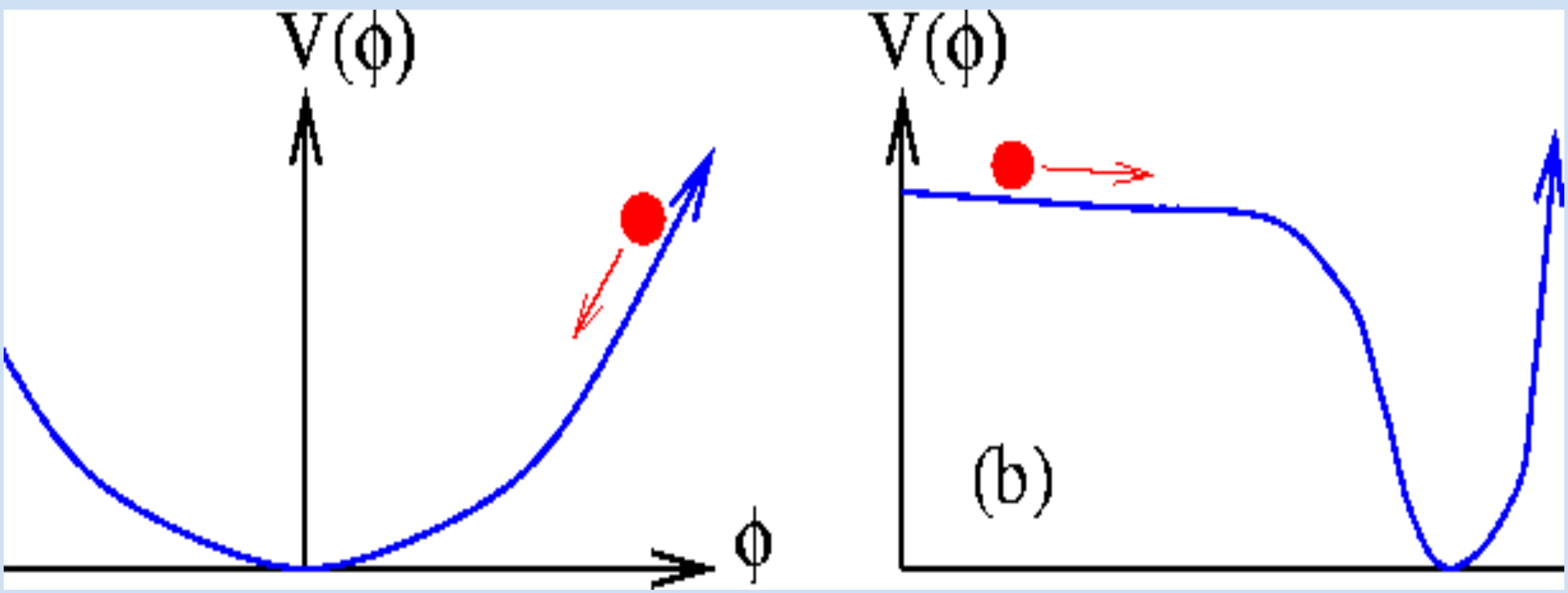


Inflation:



10^{40}
10^{20}
1
10^{-20}
10^{-40}
10^{-60}
Radius of Observed Universe (meters)
$\times 10$
Standard Theory
Inflationary Theory
Inflationary Period
10^{-40}
10^{-30}
10^{-20}
10^{-10}
1
10^{10}
Present
Time (seconds)

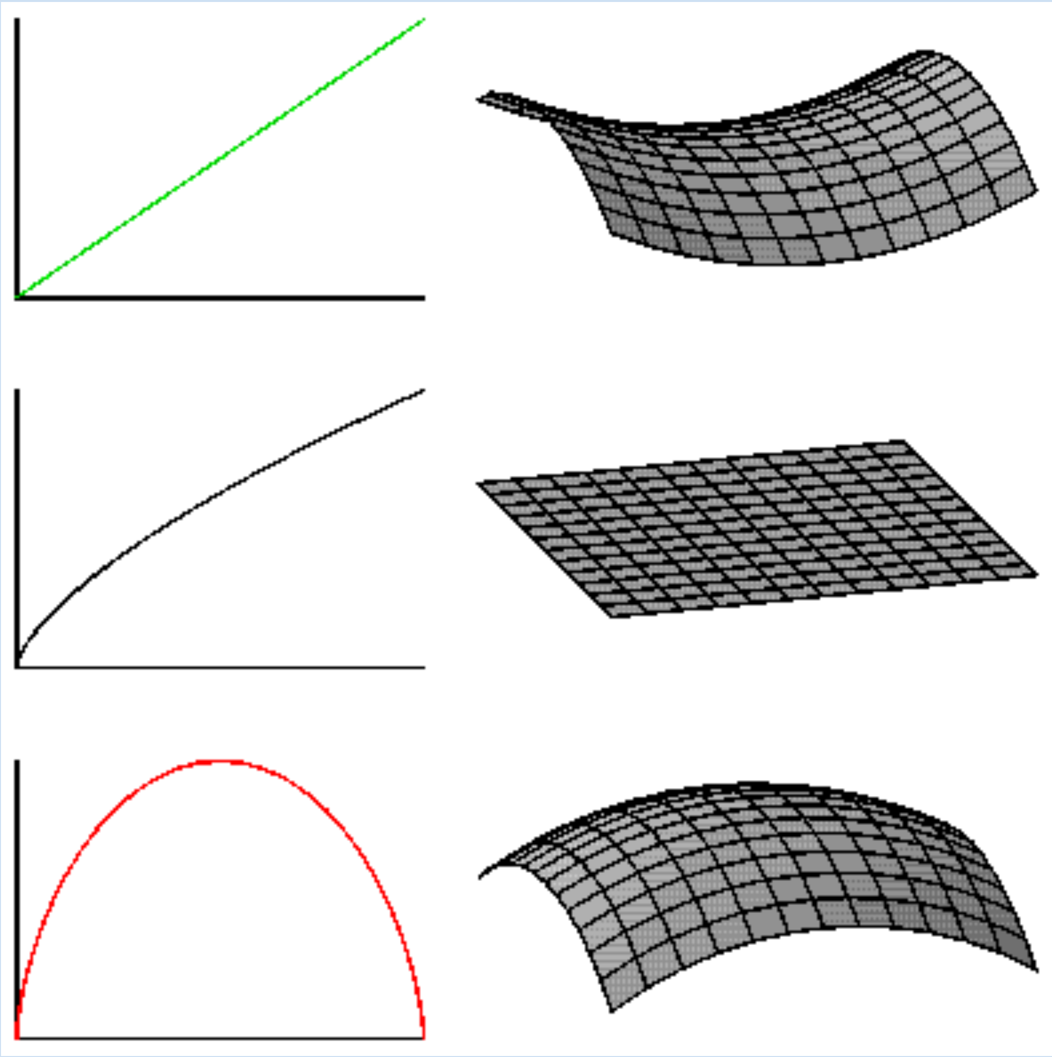
The mechanism: Vacuum energy associated with new ultra-high-energy physics (e.g., grand unification, strings, supersymmetry, extra dimensions….)



Inflation prediction #1:
The Universe is flat

Cosmological geometry: The shape of spacetime

General relativity: Matter warps spacetime



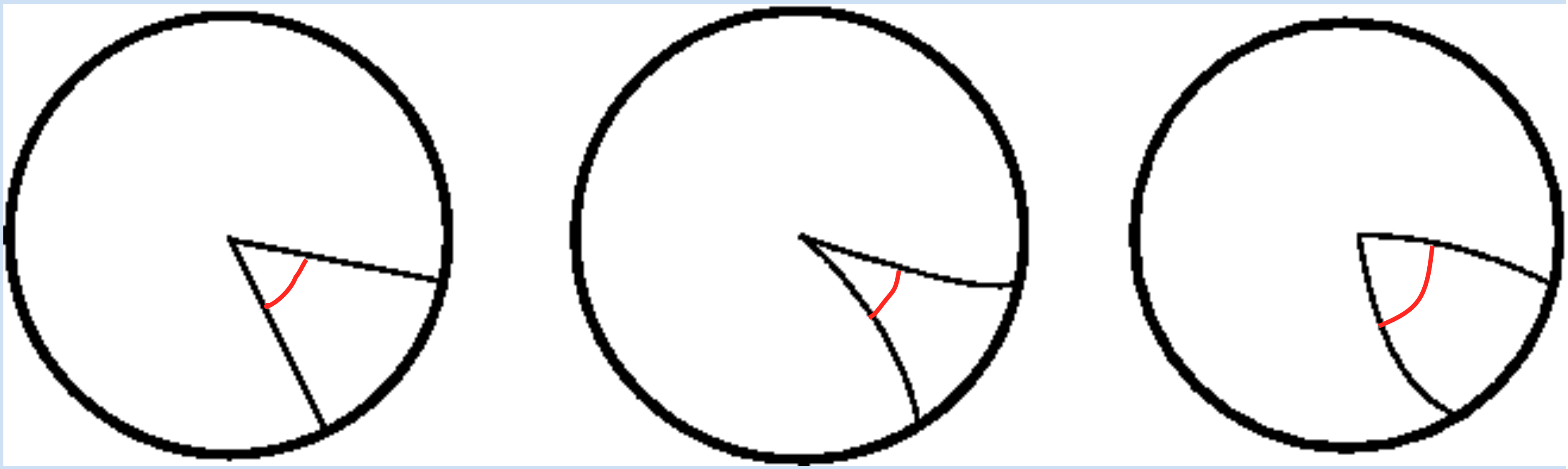
"Open"
(Less matter)

"Flat"
(critical density)

"Closed"
(more matter)

The Geometry of the Universe

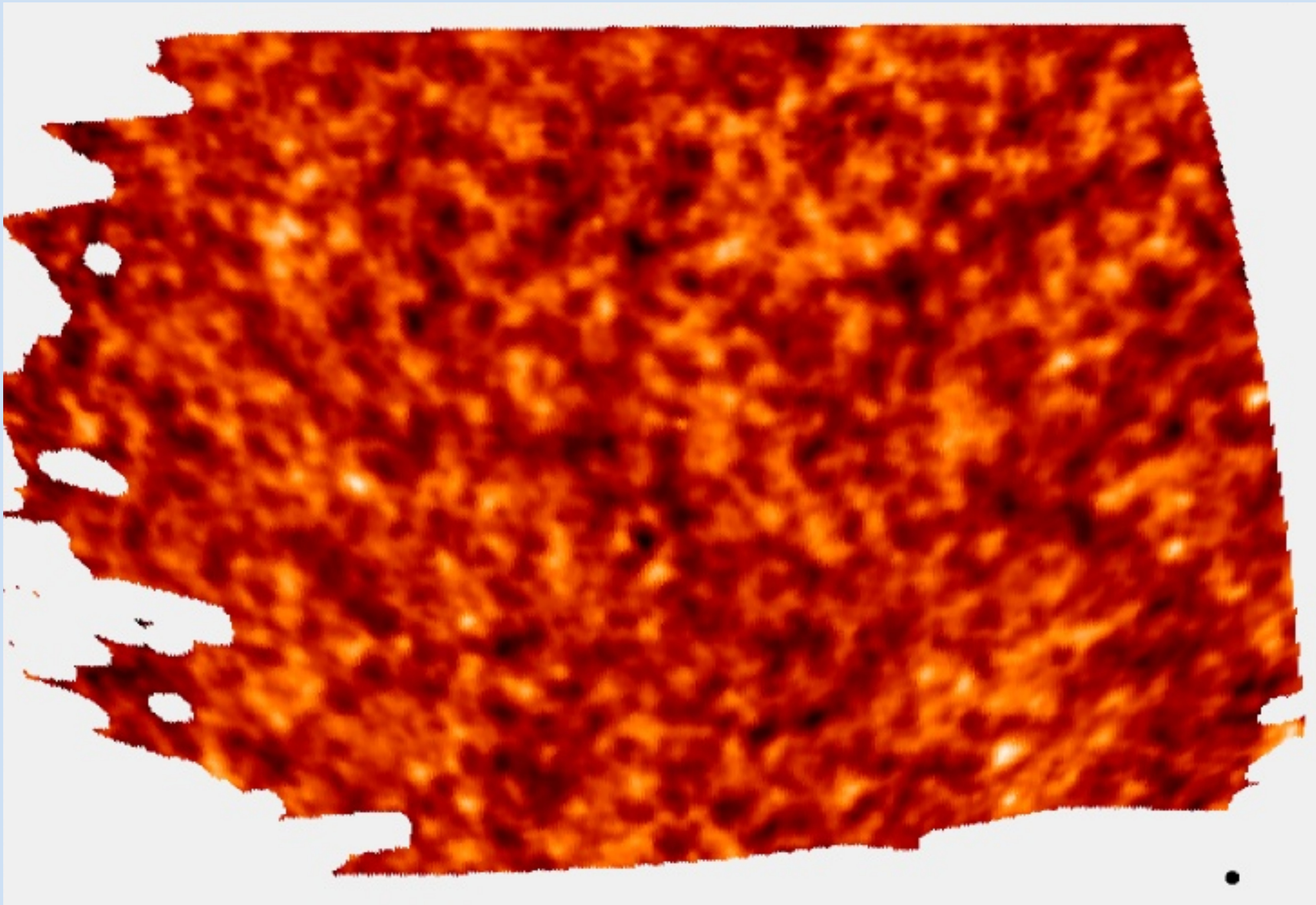
Warped spacetime acts as lens:



"flat" "open" "closed"

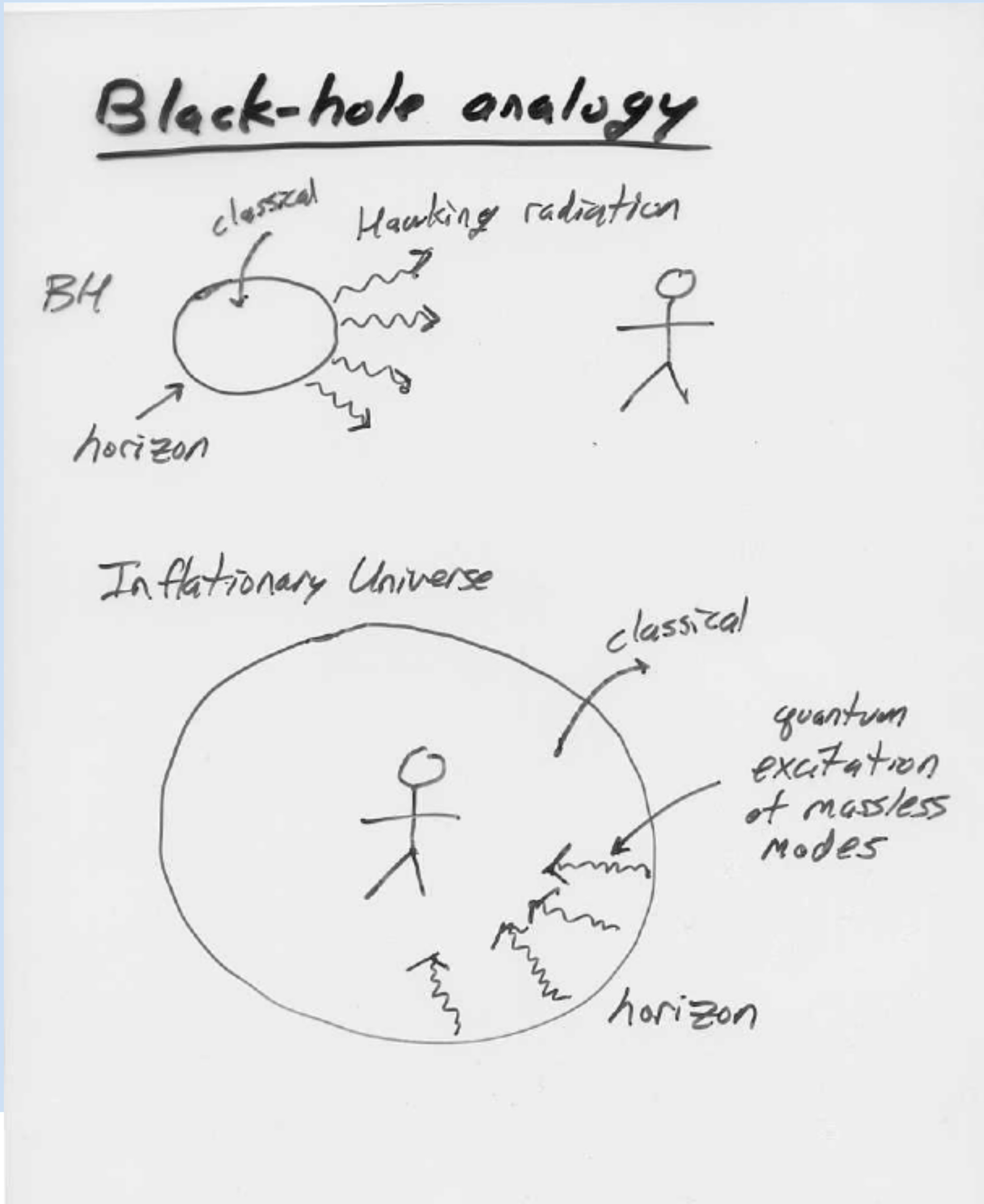
(MK, Spergel, Sugiyama 1994)

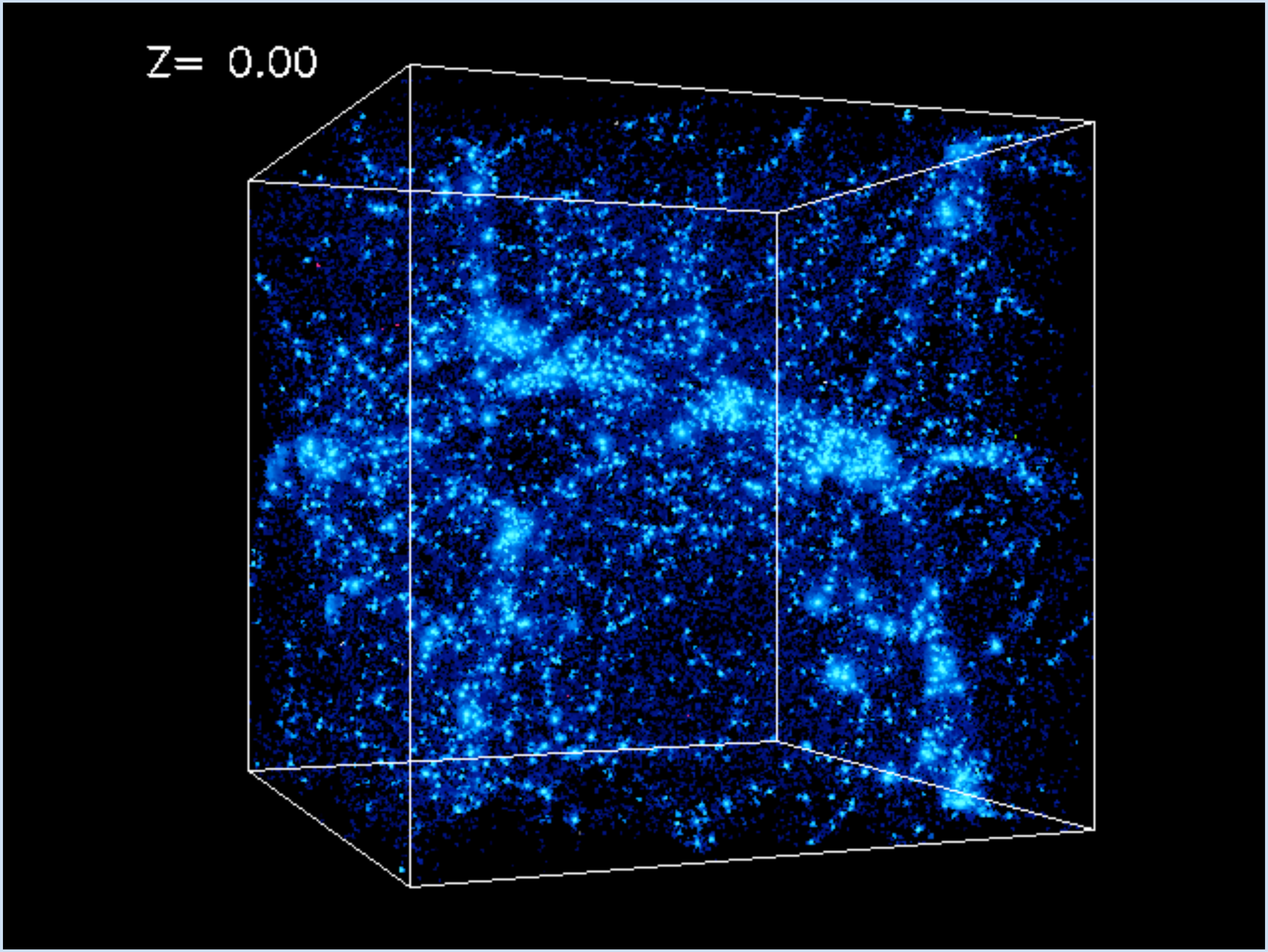
Map of CMB (Boomerang/MAXIMA 2000)



Sizes of hot/cold spots $\Rightarrow$ Universe is flat

Inflation prediction #2: Primordial density perturbations




Z= 0.00

Inflation predicts power spectrum

$$P(k) \equiv \left\langle \left(\frac{\delta\rho}{\rho} \right)^2_{\vec{k}} \right\rangle \propto k^{n_s}$$

With

$$n_s = 1 - 2\epsilon + 6\eta$$

Recent experimental results:

$$n_s \simeq 0.95$$

$$n_s \neq 1$$

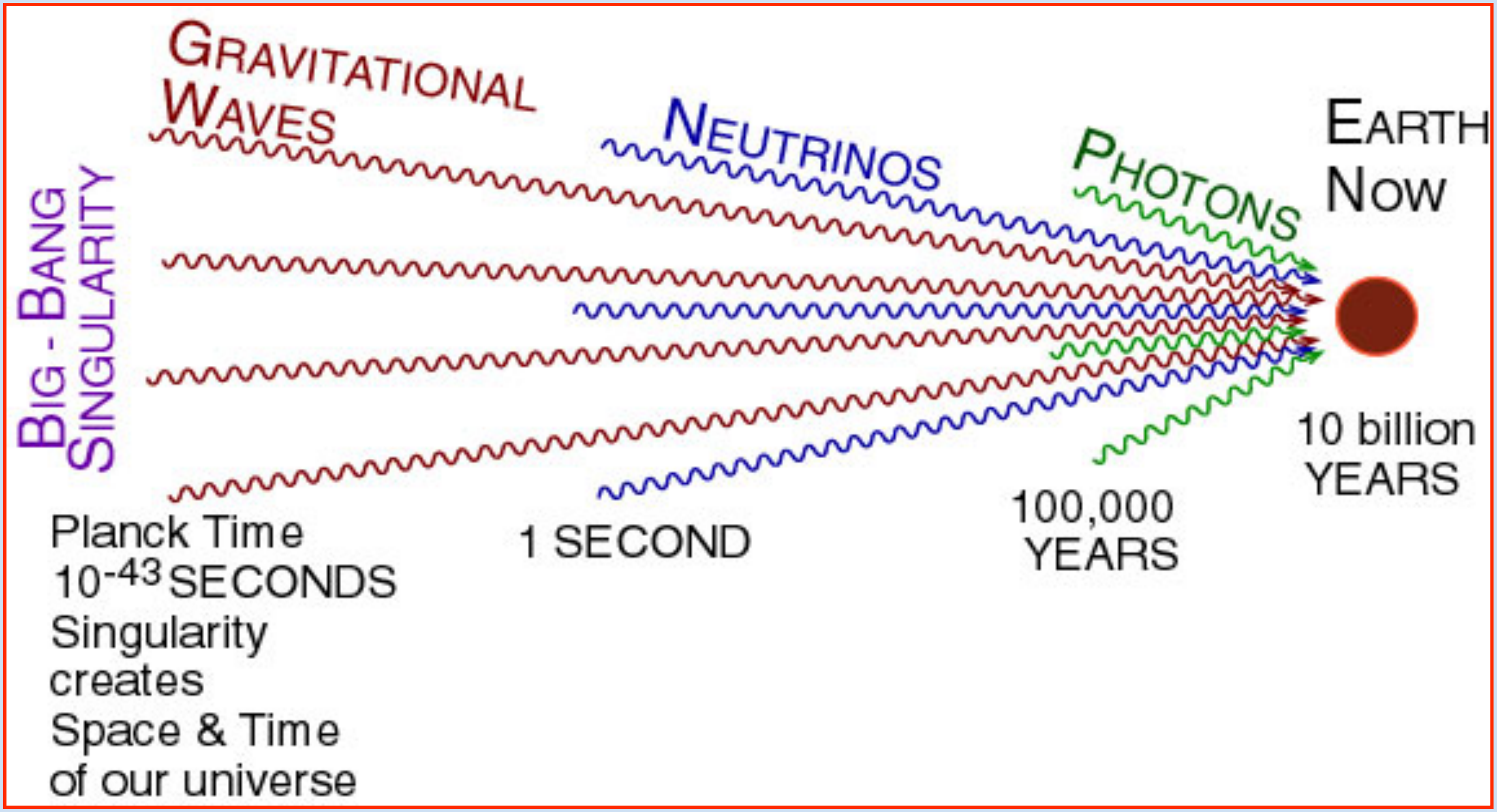
At ~3-sigma level

WHAT NEXT???

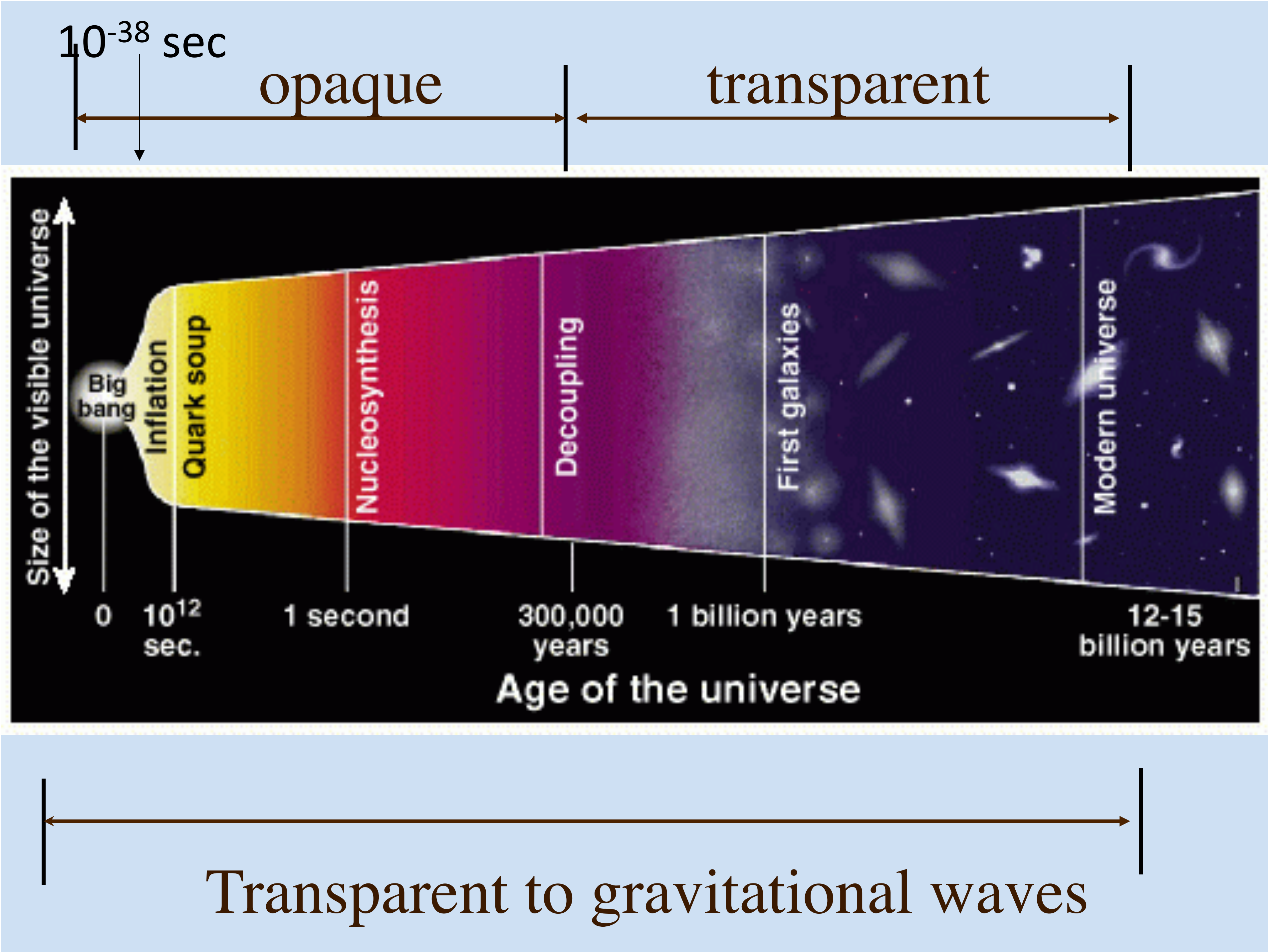
What is new physics responsible for inflation?

What is $V(\phi)$??

STOCHASTIC GRAVITATIONAL WAVE BACKGROUND with amplitude $\propto V^{1/2}$



GRAVITATIONAL WAVES
NEUTRINOS
PHOTONS
EARTH NOW
BIG - BANG SINGULARITY
Planck Time
10-43 SECONDS
Singularity creates Space & Time of our universe
1 SECOND
100,000 YEARS
10 billion YEARS



10^{-38} sec
opaque
transparent
Size of the visible universe
Big bang
Inflation
Quark soup
Nucleosynthesis
Decoupling
First galaxies
Modern universe
0
10^{12} sec.
1 second
300,000 years
1 billion years
12-15 billion years
Age of the universe
Transparent to gravitational waves

Detection of gravitational waves with CMB polarization

"E modes"

Temperature map: $T(\hat{n})$

"B modes"

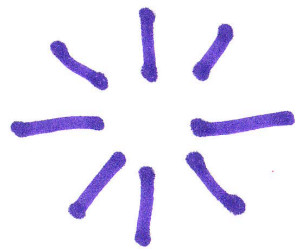
Polarization Map: $\vec{P}(\hat{n}) = \vec{\nabla} A + \vec{\nabla} \times \vec{B}$

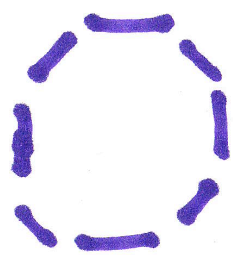
Density perturbations have no handedness
so they can*not* produce a polarization with a curl

Gravitational waves do have a handedness, so they can (and do) produce a curl

(MK, Kosowsky, Stebbins 1996; Seljak, Zaldarriaga 1996)

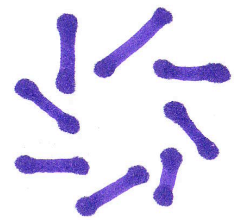
E modes

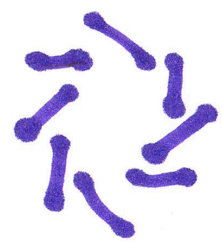




No handedness

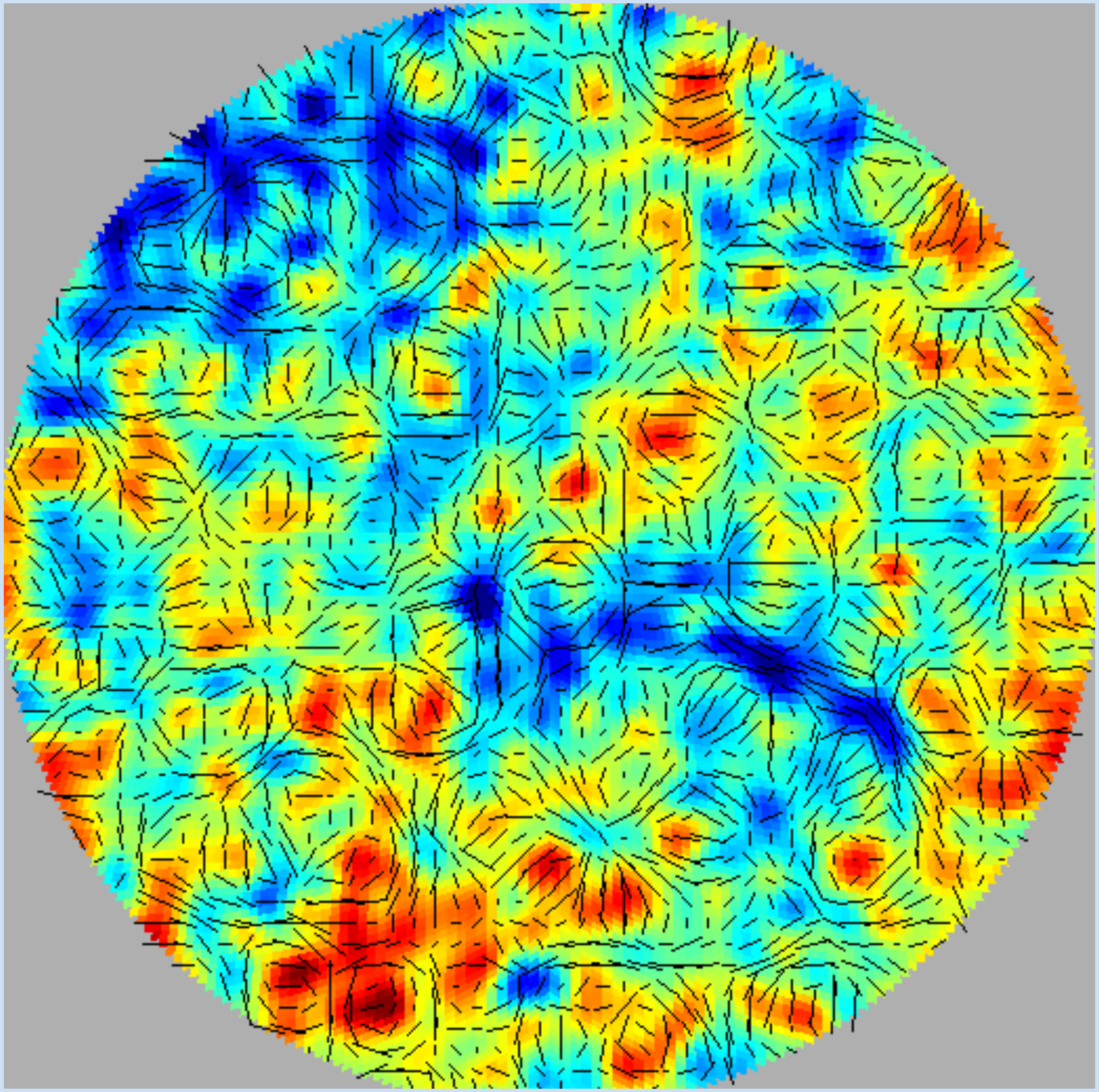
B modes



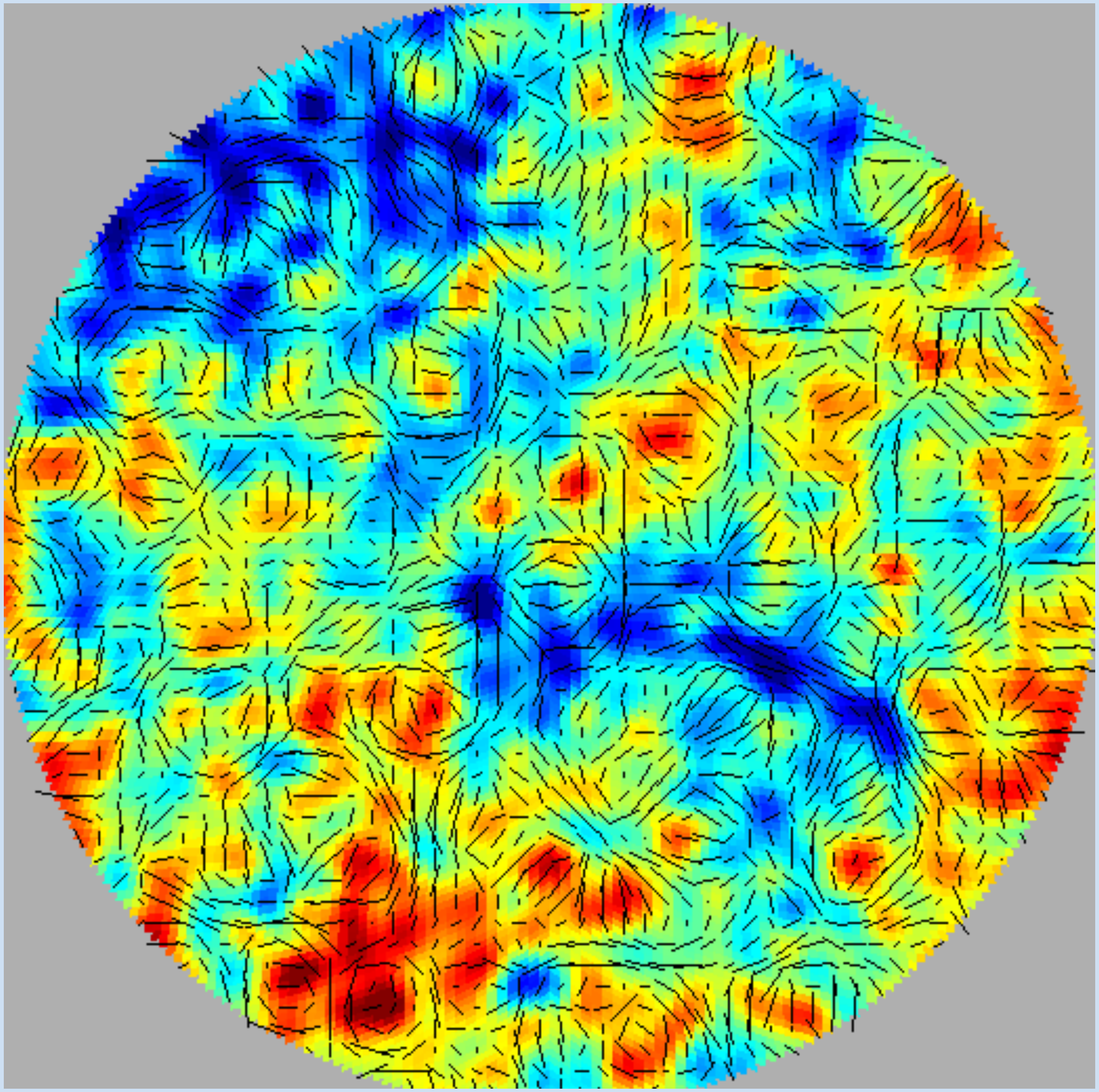


Handedness

No Gravity Waves



Gravity Waves



And one final prediction: Gaussianity

- Gravitational potential (e.g., Verde, Wang, Heavens, MK, 2000)

$$\Phi = \phi + f_{\rm NL}\phi^2$$

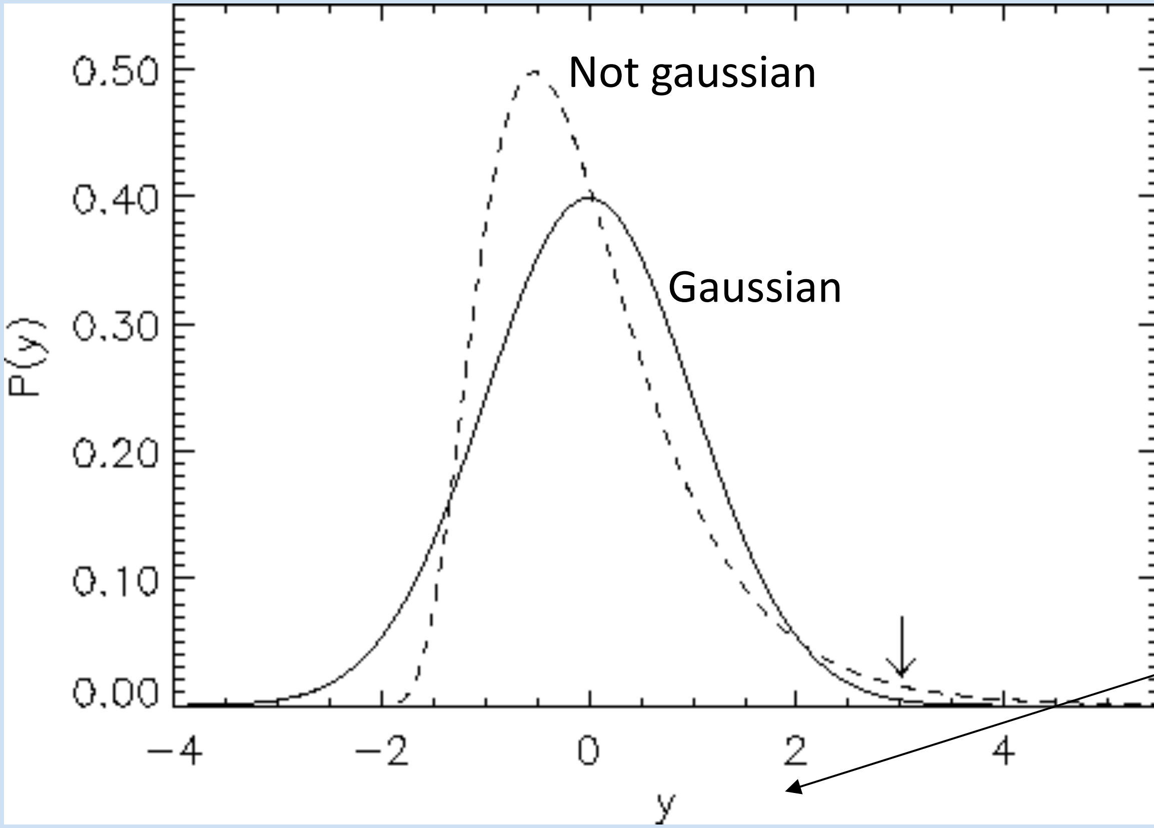
Gaussian field

with f_{NL}<1 (e.g., Wang & MK, 2000)

Fractional departure from Gaussianity:

$$\sim f_{\rm NL}\phi_{\rm rms} \sim 10^{-3}\, f_{\rm NL}$$

f_{NL}~5 detectable by Planck



$\Delta T/T$

Current constraints (WMAP5,SDSS):

$|f_{nl}|<100$

Inflation doing well

- Geometry
- $n_s \sim 1$
- Gaussian

But standard single-field slow-roll inflation is a toy model! It cannot be the whole story!

- Possible embeddings in new UHE physics give rise to, e.g.,
  - Funny kinetic terms ("DBI inflation")
  - Multiple fields (e.g., curvaton)
  - Wiggles/bumps/breaks in the inflaton potential (BSI models or axion monodromy)
  - Topological-defect production
  - ETC

Next steps (in progress)

- $n_s \neq 1$?
- Gravitational waves
- **non-Gaussianity**

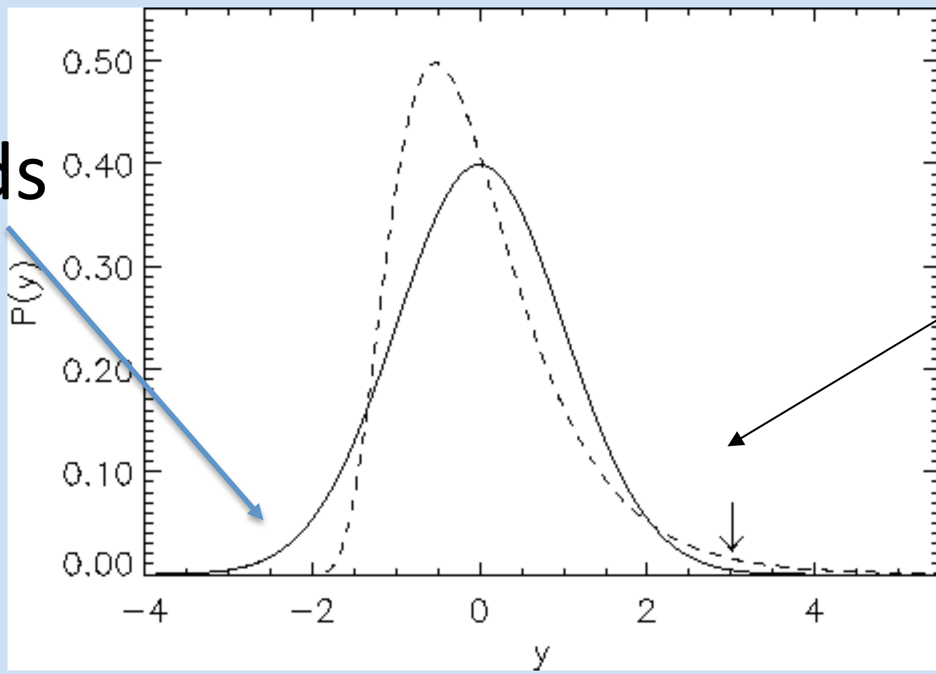
Beyond-SFSR models “predict” (“allow for”?)

- Departures from standard expectations for n_s, gravitational waves, adiabaticity.....
- A lot more (and more flavors of) non-Gaussianity

How do we tell if primordial perturbations were Gaussian??
Some earlier work

With abundances/properties of rare objects: e.g., clusters (e.g., Robinson, Gawiser & Silk 2000; Verde, MK, Mohr, Benson 2001) or high redshift galaxies (e.g., Verde, Jimenez, Matarrese, MK 2001) or voids (MK, Verde, Jimenez 2009)

Voids



Rare objects form here

Non-Gaussianity beyond the PDF: The Bispectrum

- The gravitational potential $\Phi(\vec{x})$ in early Universe has Fourier components $\Phi_{\vec{k}}$.
- Power spectrum is $P_\Phi(k) = \left\langle \left|\Phi_{\vec{k}}\right|^2 \right\rangle$
- Different modes have zero covariance:

$$\left\langle \Phi_{\vec{k}_1} \Phi_{\vec{k}_2} \right\rangle = 0 \quad \text{for} \quad \vec{k}_1 \neq \vec{k}_2$$

- Bispectrum is

$$\left\langle \Phi_{\vec{k}_1} \Phi_{\vec{k}_2} \Phi_{\vec{k}_3} \right\rangle = B(k_1, k_2, k_3)\delta_{\vec{k}_1+\vec{k}_2+\vec{k}_3,\vec{0}}$$

E.g.,

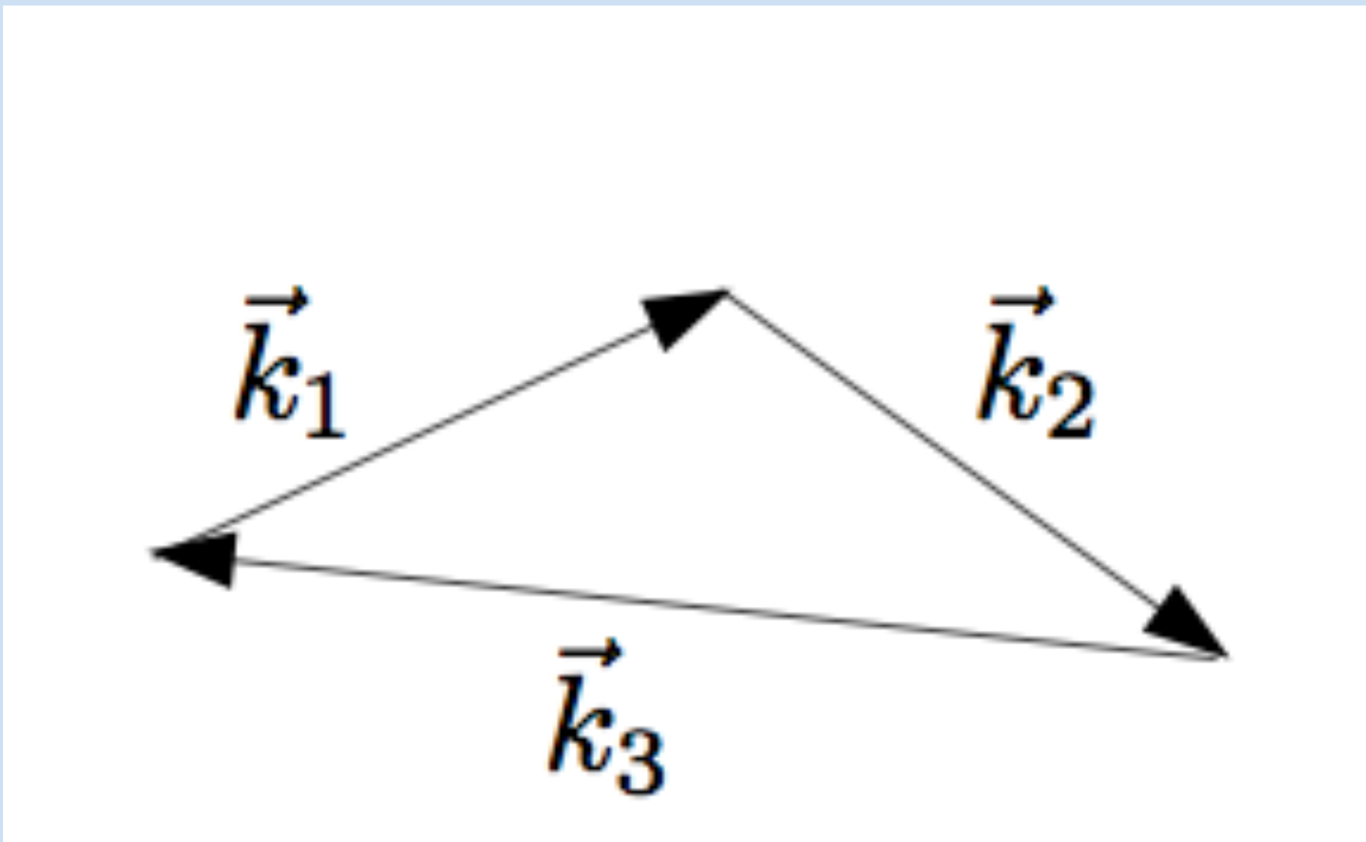
- Gaussian fluctuations: B=0
- Local model, $\Phi = \phi + f_{nl}\left(\phi^2 - \langle\phi^2\rangle\right)$
  has bispectrum

$$B(k_1, k_2, k_3) = 2 f_{nl}\left[P_\Phi(k_1)P_\Phi(k_2) + P_\Phi(k_2)P_\Phi(k_3) + P_\Phi(k_1)P_\Phi(k_3)\right]$$

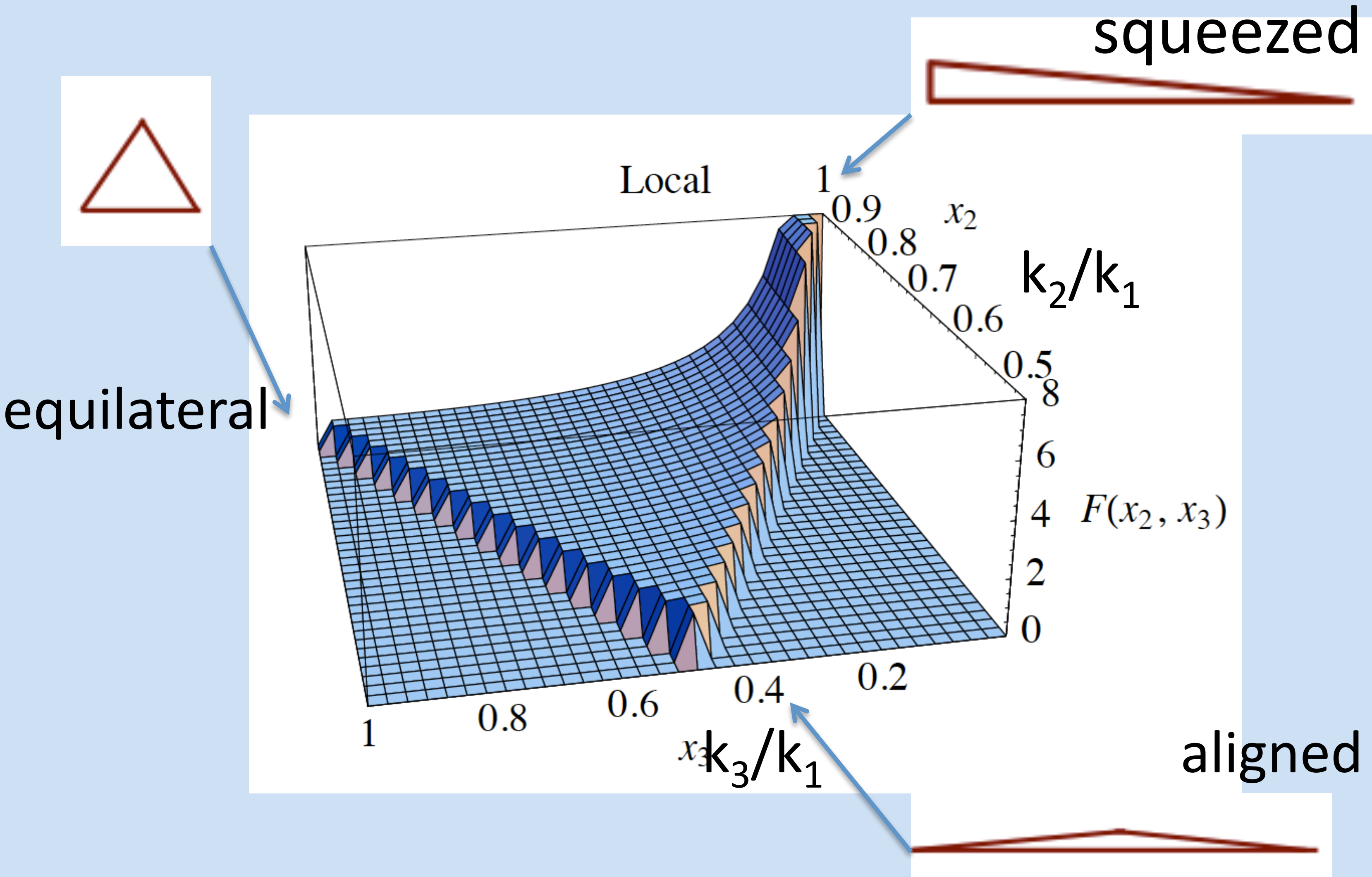
or for scale-invariant spectrum,

$$B(k_1, k_2, k_3) = 2 f_{nl}\left[\frac{1}{k_1^2 k_2^2} + \frac{1}{k_2^2 k_3^2} + \frac{1}{k_1^2 k_3^2}\right]$$

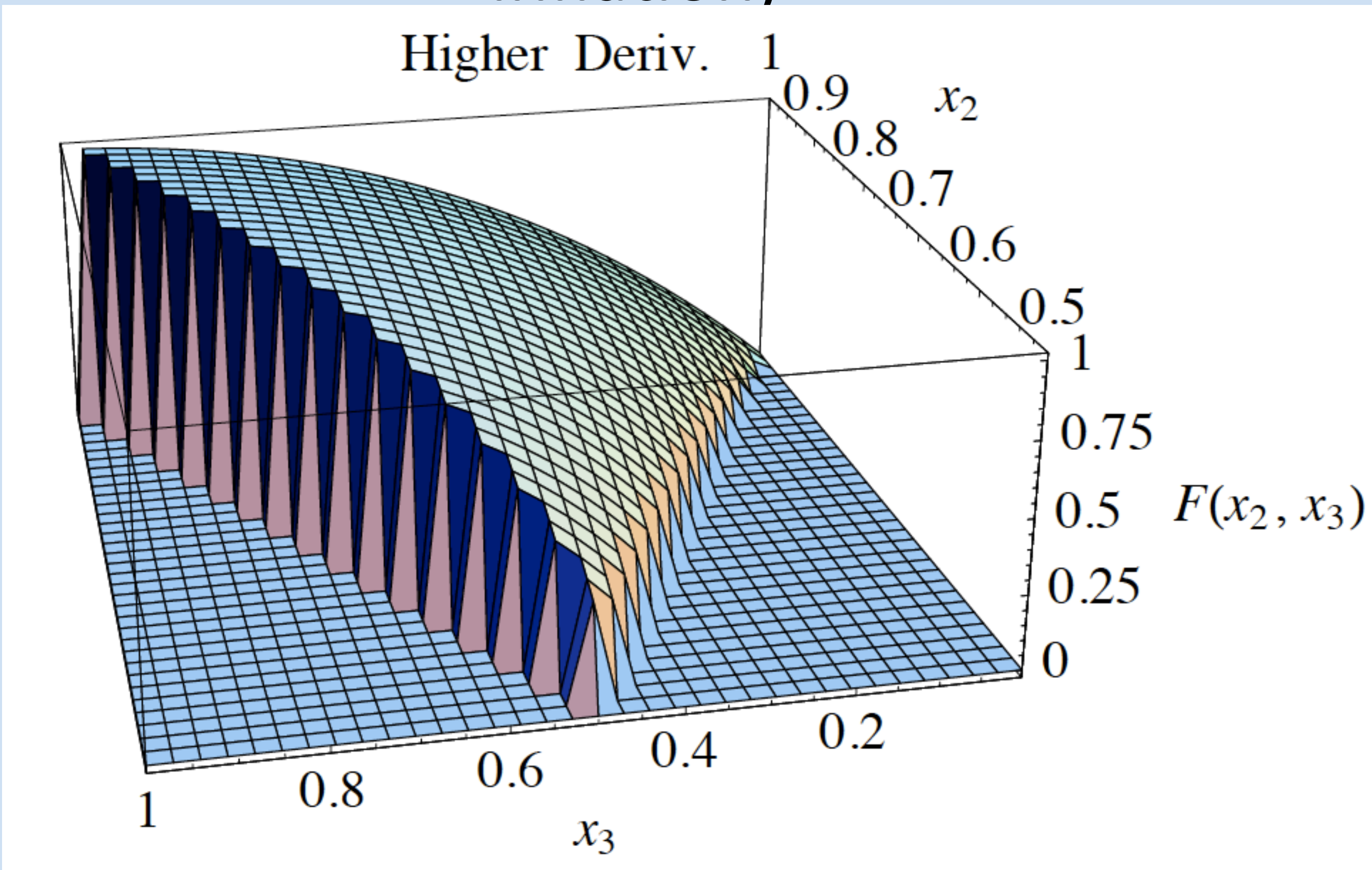
Bispectrum is function of triangle shape:



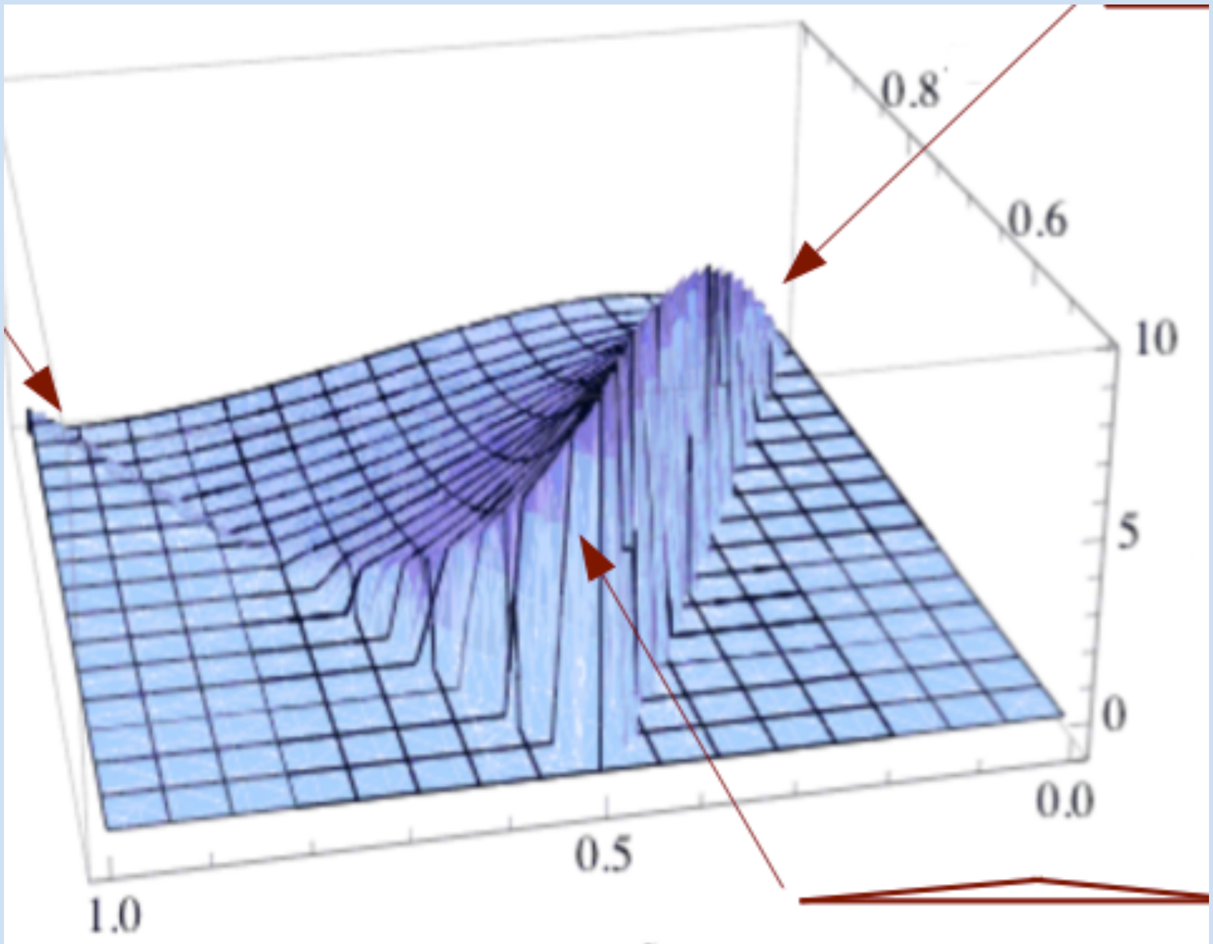
For local model,



But there are other possibilities; e.g., "equilateral" model (from DBI inflation)



Or the "orthogonal" model (from some single-field funny-vacuum theories



Self-Ordering Scalar Fields: Another possibility for new beyond-SFSR physics (Figueroa, Caldwell, MK 2010)

- Consider N-component scalar field,

$$\vec{\Phi} = (\phi^1, \phi^2, \cdots, \phi^N)$$

with Lagrangian

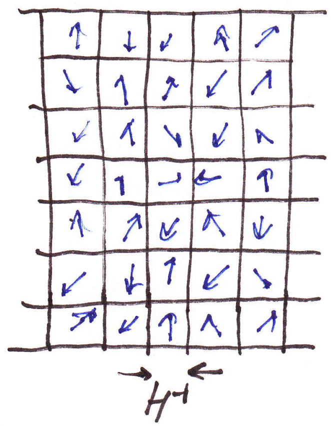
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \vec{\Phi}) \cdot (\partial^\mu \vec{\Phi}) - V(|\vec{\Phi}|)$$

with O(N) symmetry and potenial

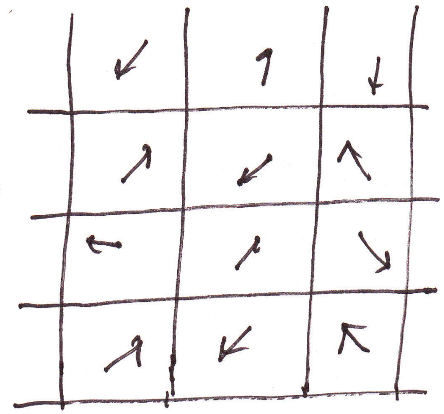
$$V(\vec{\Phi}) \propto (|\Phi|^2 - v^2)^2$$

- At low T (after SSB), $|\vec{\Phi}| = v$, but $\vec{\Phi}$ points in different direction in vacuum manifold $S^{N\text{-}1}$ in each Hubble volume
- As different regions come in causal contact, find energy densities $|\nabla\vec{\Phi}|^2/2 \sim H^2 v^2$ or fractional density perturbations

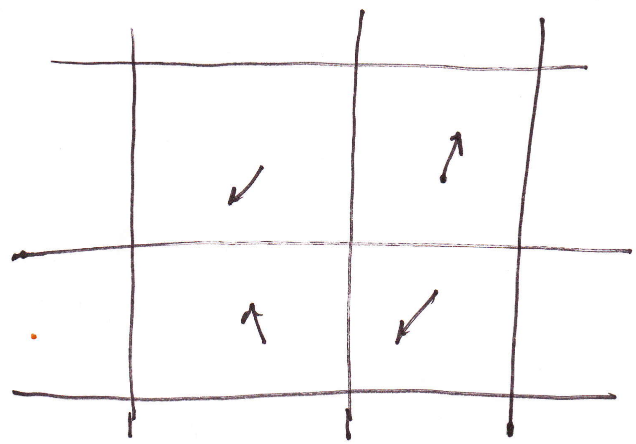
$$\frac{\delta\rho}{\rho} \sim \frac{H^2 v^2}{H^2 M_{Pl}^2} \sim \left(\frac{v}{M_{Pl}}\right)^2 \sim 10^{-6}\left(\frac{v}{10^{16}\,\mathrm{GeV}}\right)^2$$



Early times



later time



even later

- Perturbations are scale-invariant
- Are ~isocurvature (but incoherent), *not* adiabatic
- Are *highly* non-Gaussian:
  in large-N limit, ϕ^i are ~Gaussian, but $\delta\rho \sim \Phi^2 \sim (\text{Gaussian})^2$
- Observationally: Can*not* be *the* primordial perturbations, but may account for ~10% of primordial perturbations
- ~10% contribution to primordial perturbations if GUT scale!

Analytical calculation of bispectrum for this model (Figueroa, Caldwell, MK, 2010, extending work of Jaffe 1994)

First: <10% contribution to measured C_ls:

$$\frac{v}{N^{1/4}} \lesssim \frac{M_{Pl}}{2000}$$

Bispectrum is

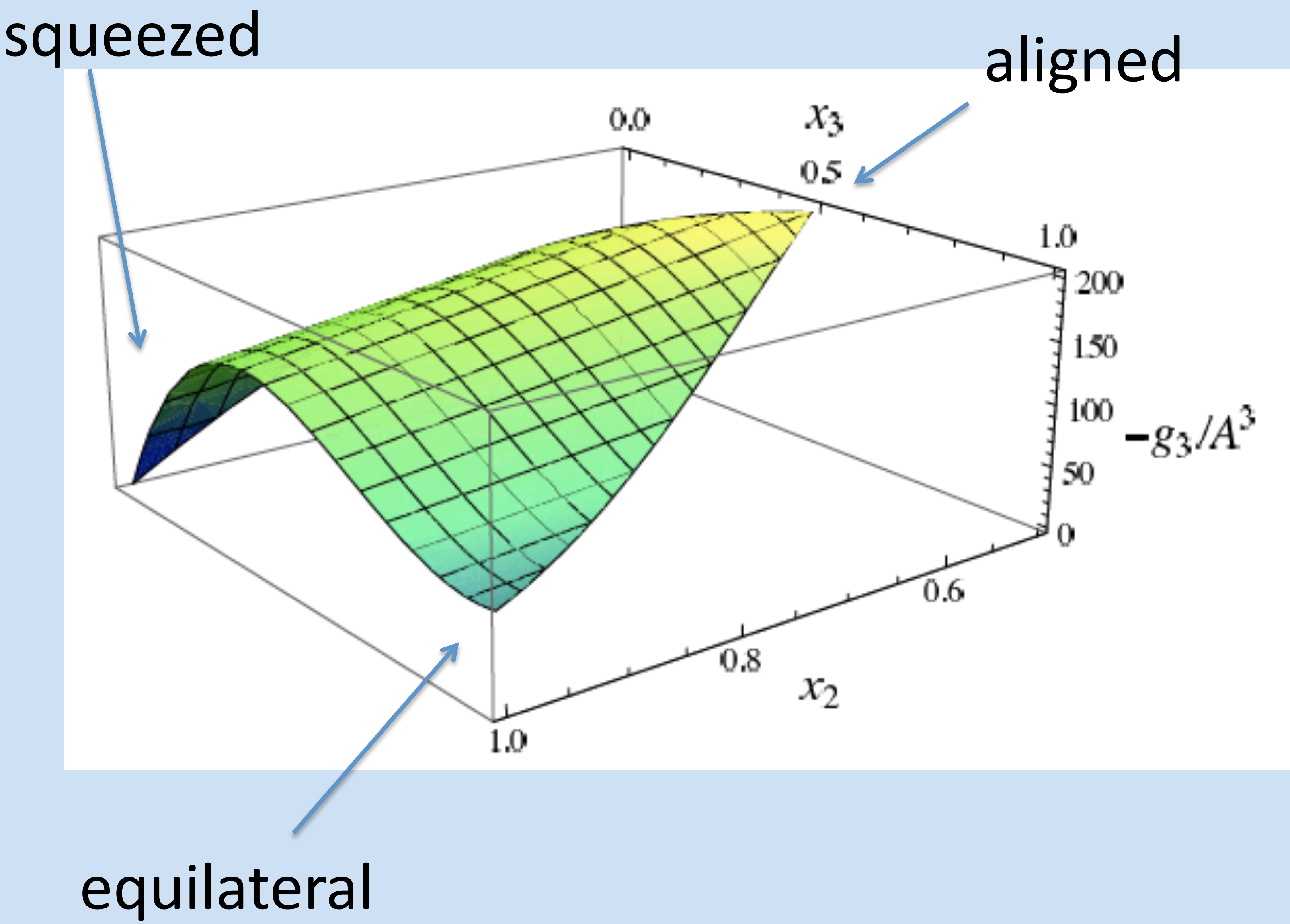
$$B(k_1, k_2, k_3) = \left(\frac{2\pi}{5} \frac{v}{M_{Pl}} \frac{\eta^2}{A} \right)^3 \frac{1}{N^2} g_3(k_1, k_2, k_3)$$

With (Jaffe 1994)

$$g_3(\mathbf{v}) = \int d^3u\, \mathbf{u} \cdot \hat{\mathbf{k}}\, I(|\hat{\mathbf{k}} - \mathbf{u}|, u) \Big[(1 + 2\mathbf{v} \cdot \hat{\mathbf{k}} - 2\mathbf{u} \cdot \hat{\mathbf{k}} + v^2 - 2\mathbf{u} \cdot \mathbf{v})(2\mathbf{u} \cdot \mathbf{v} - 2\mathbf{v} \cdot \hat{\mathbf{k}} - v^2)$$
$$\times I(|\hat{\mathbf{k}} + \mathbf{v} - \mathbf{u}|, |\mathbf{u} - \hat{\mathbf{k}}|) I(u, |\hat{\mathbf{k}} + \mathbf{v} - \mathbf{u}|)$$
$$+ (v^2 + 2\mathbf{u} \cdot \mathbf{v})(2\mathbf{u} \cdot \hat{\mathbf{k}} + 2\mathbf{u} \cdot \mathbf{v} + v^2 - 1) I(u, |\mathbf{u} + \mathbf{v}|) I(|\mathbf{u} - \hat{\mathbf{k}}|, |\mathbf{v} + \mathbf{u}|) \Big],$$

which we re-write

$$g_3(k_1, k_2, k_3) \equiv \int \frac{d^3 v}{(2\pi)^3} H(\mathbf{u} + \mathbf{v}, \mathbf{v}) H(\mathbf{v}, \hat{\mathbf{z}} - \mathbf{v}) \times H(\hat{\mathbf{z}} - \mathbf{v}, \mathbf{u} + \mathbf{v}),$$



squeezed
aligned
equilateral
0.0
x_3
0.5
1.0
200
150
100
50
0
$-g_3/A^3$
0.6
0.8
x_2
1.0

For those who want to do analyses:

$$g_3(k_1, k_2, k_3) = -\frac{A^3}{143}\left(262 - 127\frac{k_2}{k_1}\right)$$
$$\times\left[947\frac{k_3}{k_1} - 1770\left(\frac{k_3}{k_1}\right)^2 + 893\left(\frac{k_3}{k_1}\right)^3\right],$$

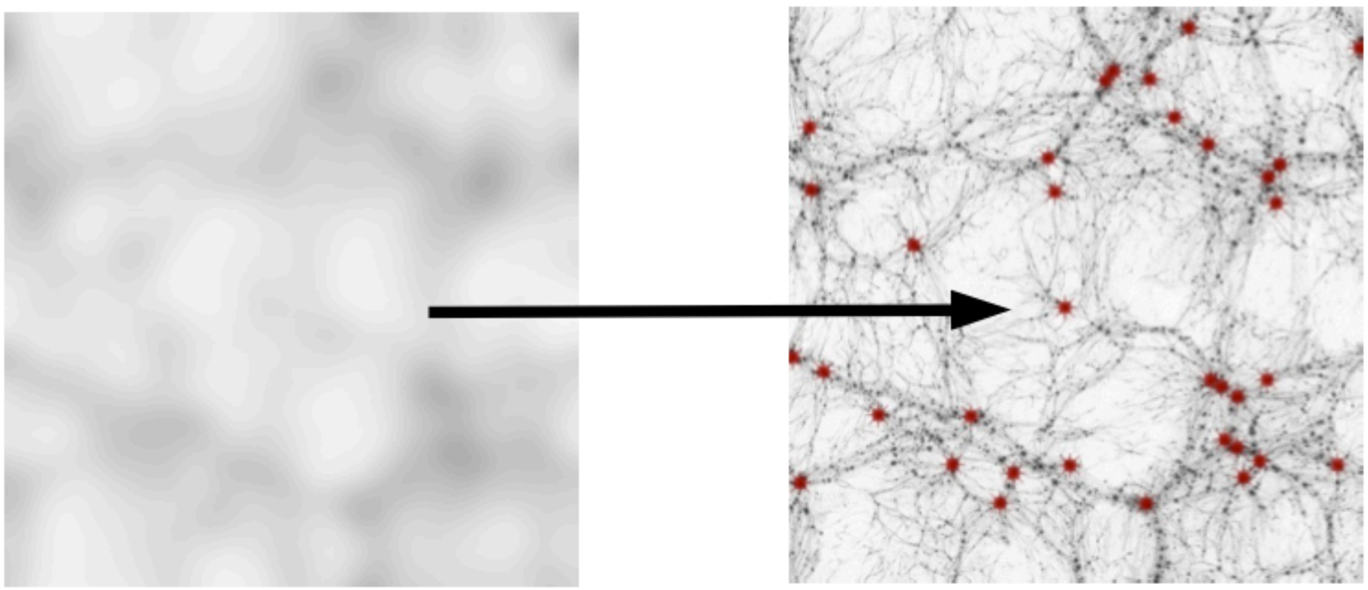
Quantitatively:

Current NG limits are not constraining, but effects should be detectable with Planck if v near current upper limits

Halo Clustering: a powerful new probe of non-Gaussianity (e.g., Dalal, Dore, Huterer, Shirokov, 2007; Slosar, Hirata, Seljak, Ho, Padmanabhan 2008; Verde, Matarrese 2008; Schmidt, MK, 2010)

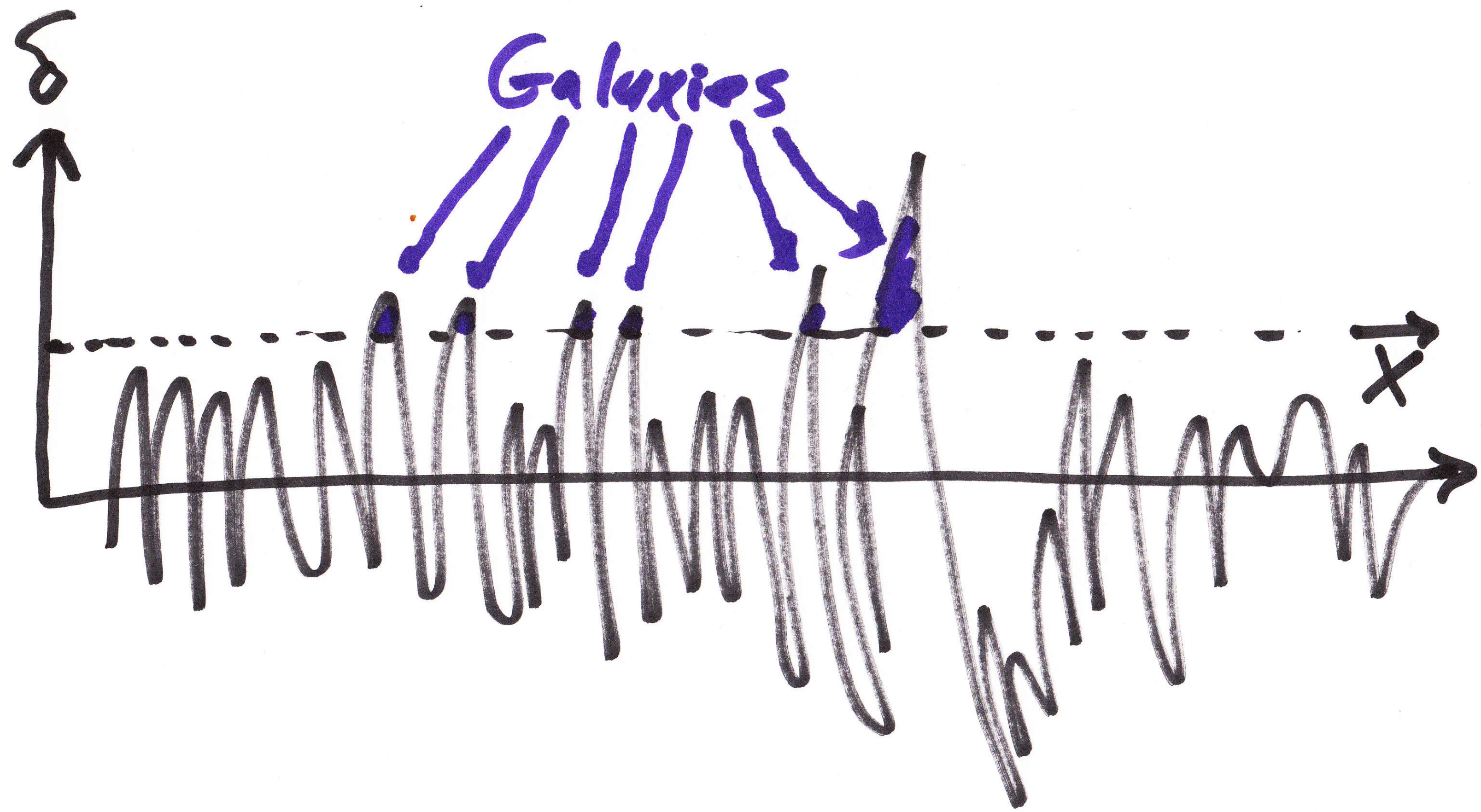
Large Scale Structure

- Observe a LSS tracer

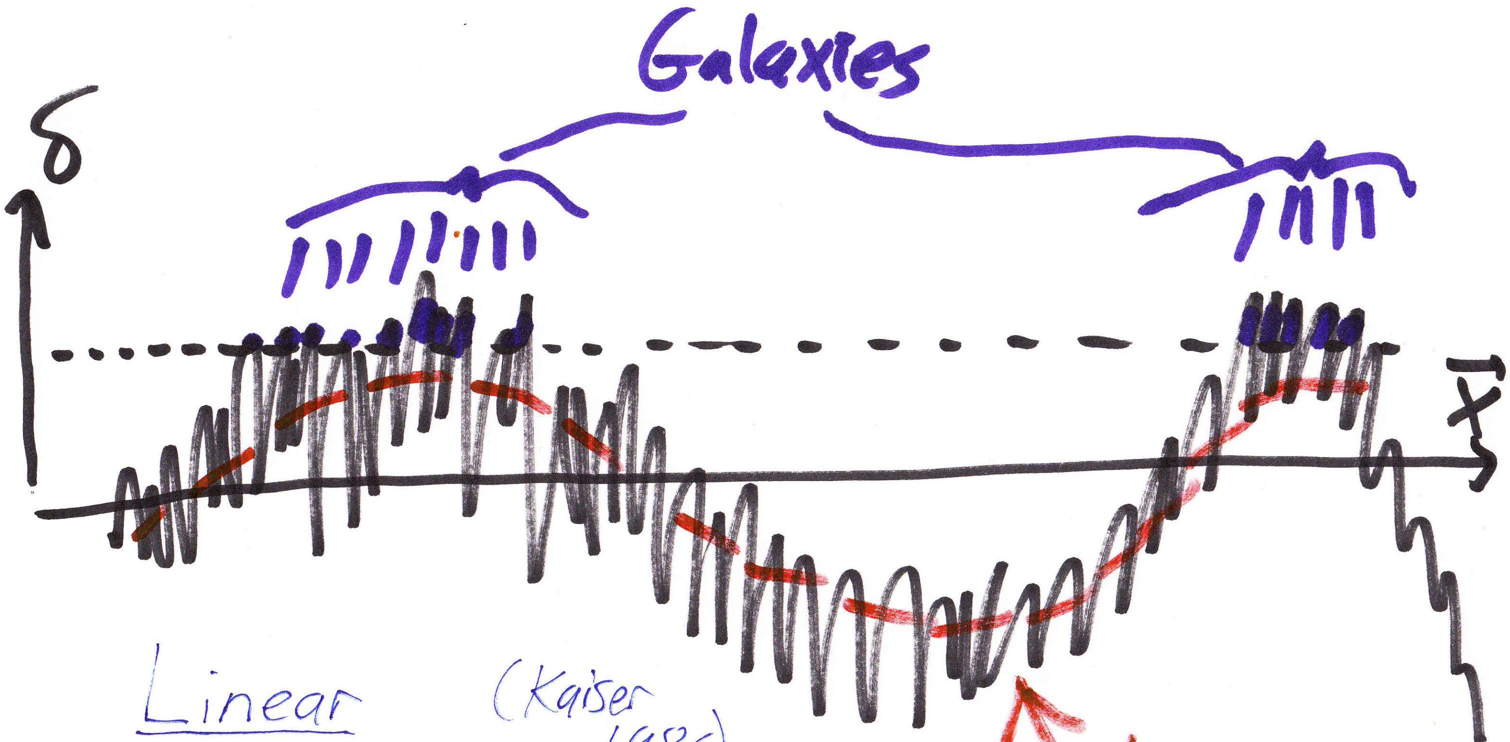


(dramatization)

- Key theoretical problem:
  - how to map *initial linear fluctuations* to observed *non-linear density field* of tracer (on large scales)



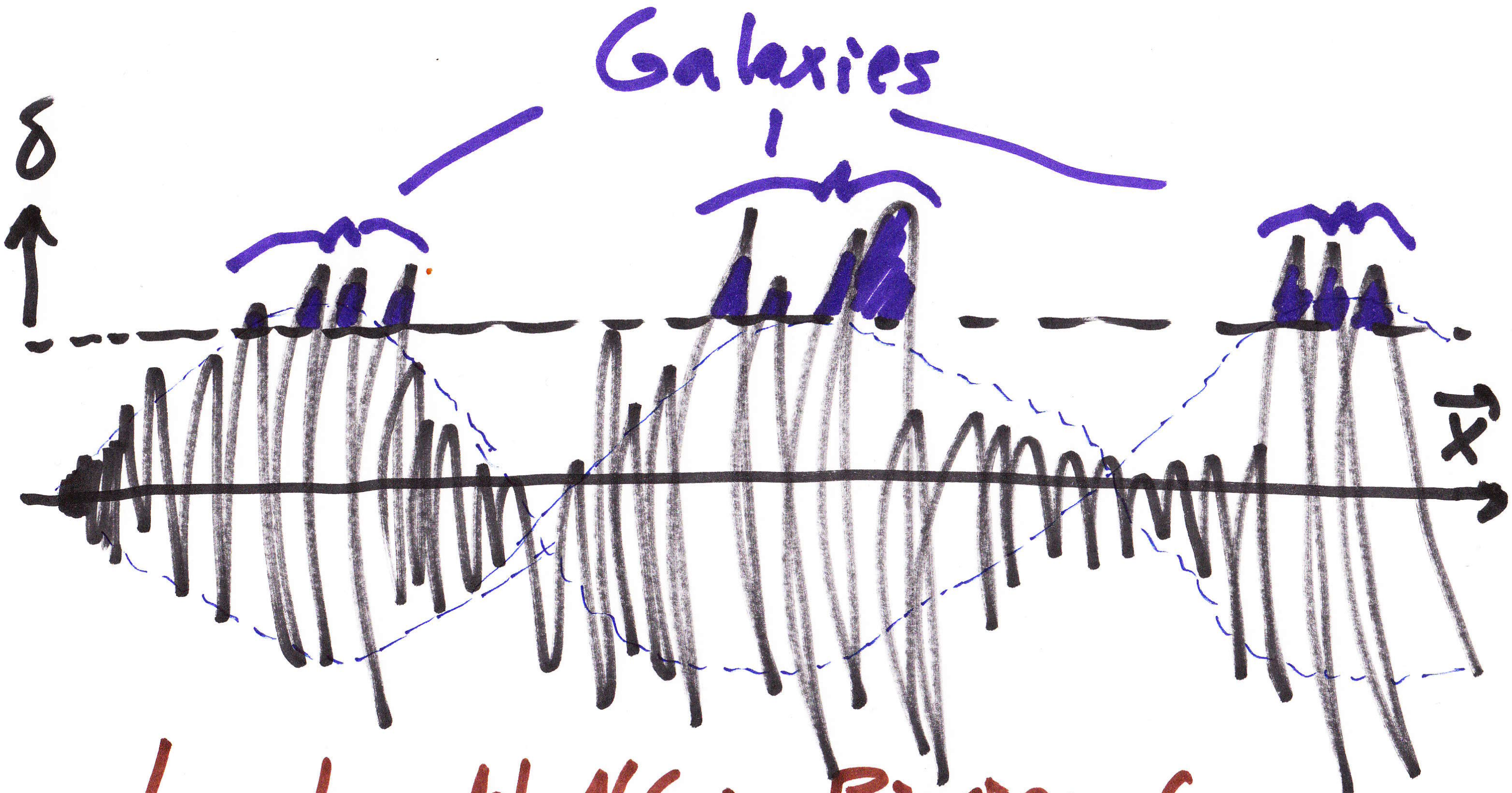
Galaxies
δ
$\vec{x}$



Long-λ Fluctuation

Linear Biasing (Kaiser 1986)

$$\frac{\delta n_g}{n_g} = b \frac{\delta \rho}{\rho}$$



Local-model NG: Biasing from modulation of Variance $\langle \delta^2 \rangle$ with $\vec{x}$

In equations: Bias for long-wavelength fluctuation of wavenumber *k*:

$$
\begin{aligned}
b_L(M,z;k) &\equiv \frac{\delta\tilde{n}(\vec{k})/n}{\delta\rho/\rho} = \frac{d\ln\tilde{n}(\vec{k})}{d\tilde{\delta}_l(\vec{k})} \\
&= \frac{\partial\ln\tilde{n}(\vec{k})}{\partial\tilde{\delta}_l(\vec{k})} + \sum_{\vec{k}_s} \frac{\partial\ln\tilde{n}(\vec{k})}{\partial P(k_s)}\frac{\partial P(k_s)}{\partial\tilde{\delta}_l(\vec{k})}
\end{aligned}
$$

Usual Gaussian bias

Non-Gaussian contribution

Use Peak-Background Split:

Separate density field into short-wavelength modes (<Mpc; halos formed at peaks) and long-wavelength modes (>Mpc; determine clustering)

Halo abundance is

$$n = n\Big(M, z; \bar{\rho}\left[1 + \delta_l(\vec{x})\right], P\big(k_s, \delta_l(\vec{x})\big)\Big)$$

Non-Gaussian contribution arises from coupling of long- and short-wavelength modes:

$$\delta_s(\vec{x}) = \delta_{g,s}(\vec{x}) + 2f_{\rm nl}\phi_{g,l}(\vec{x})\delta_{g,s}(\vec{x})$$

so local power spectrum *P(k_s)* now depends on long-wavelength perturbation

Since $\nabla^2\phi = 4\pi G\rho$, we have $\phi_{\vec{k}} \propto \delta_{\vec{k}}/k^2$

Leads to bias $b_L^{\rm ng} \propto k^{-2}$; null search with SDSS constrains $|f_{\rm nl}| \lesssim 100$ (Slosar et al. 2008)

More general bispectra

Curvaton models have,

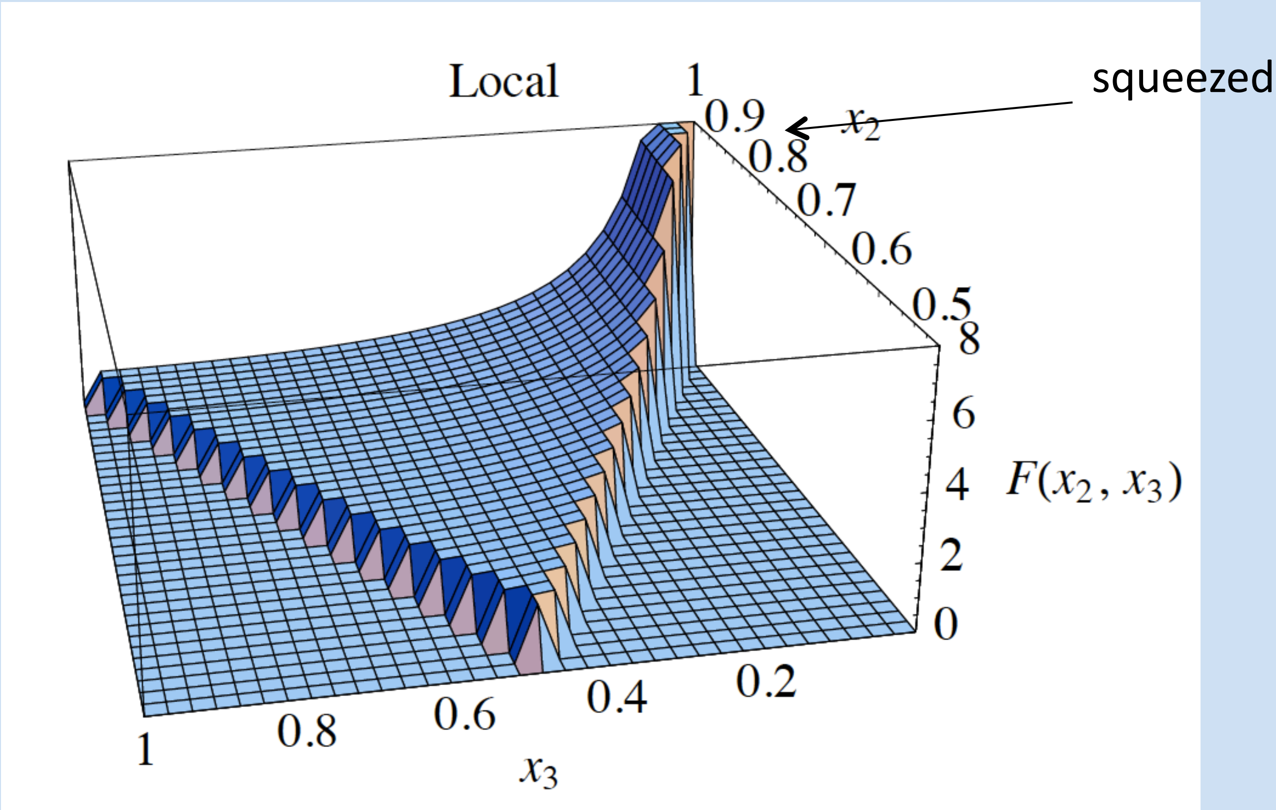
$$\Phi = \phi + f_{\rm nl}\phi^2$$

leading to bispectrum

$$\left\langle \Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3) \right\rangle = (2\pi)^3 \delta_D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B(k_1, k_2, k_3)$$

with

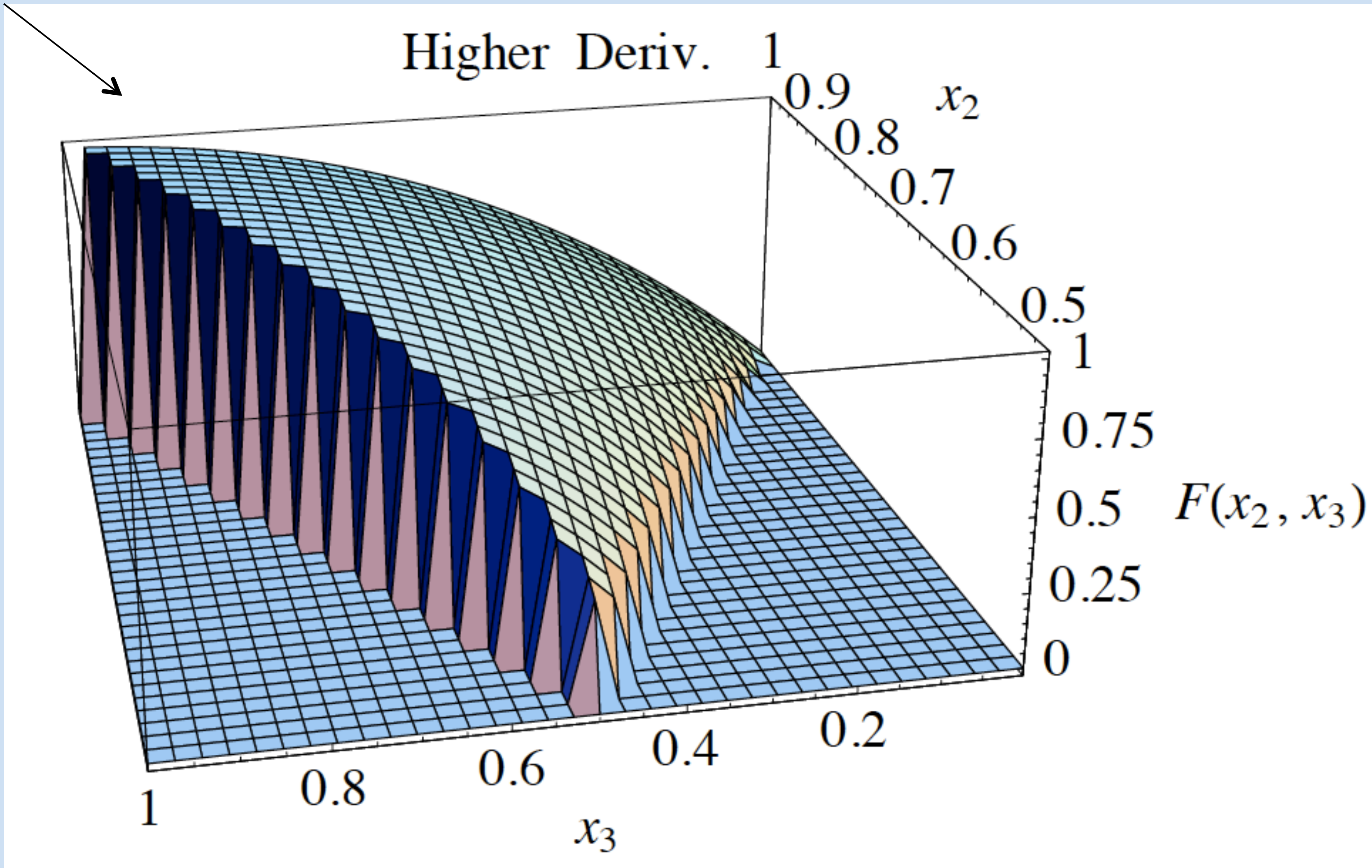
$$B(k_1, k_2, k_3) = 2 f_{\rm nl} \left[P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_1)P(k_3) \right]$$



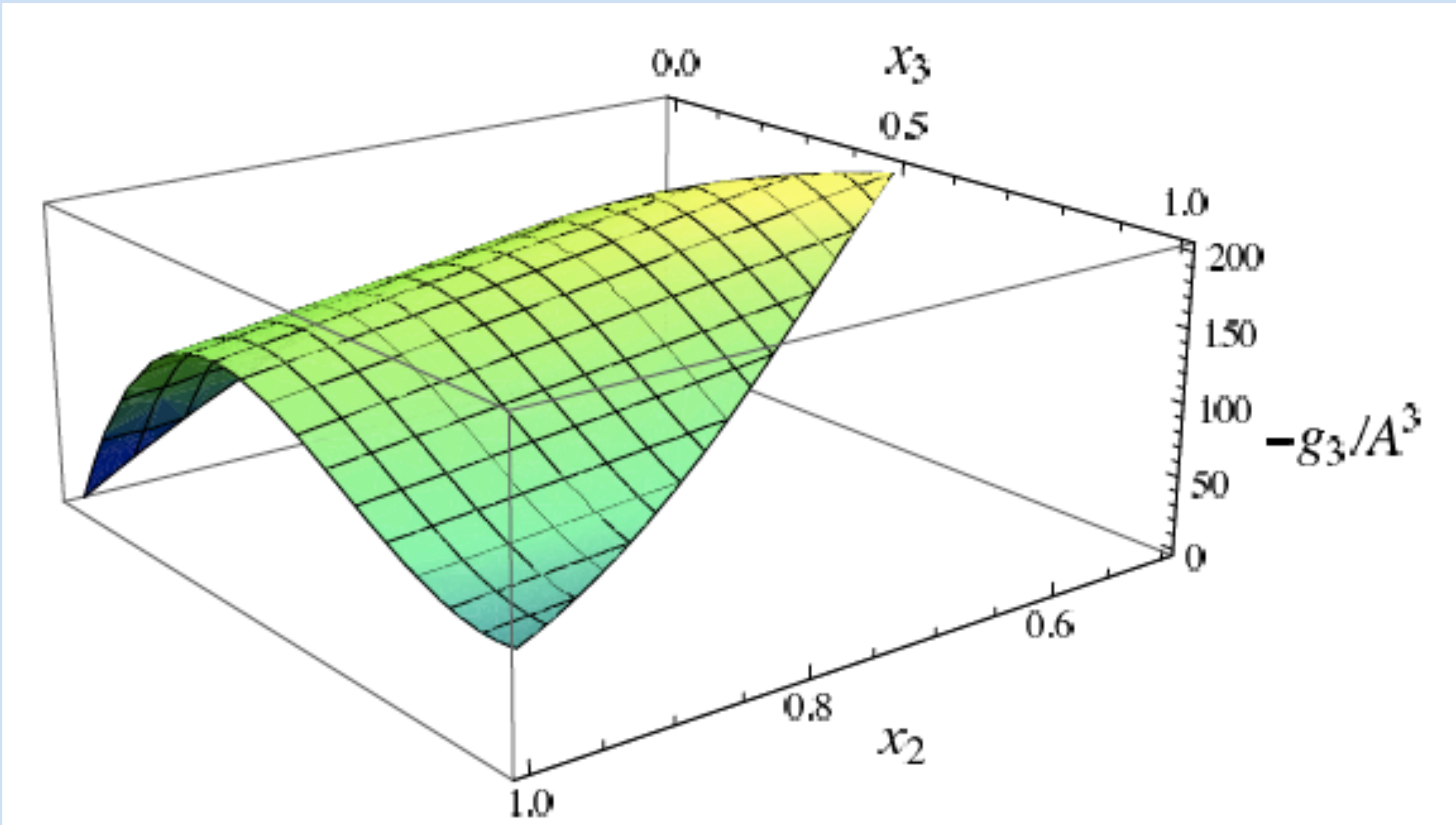
Local
squeezed
x_2
1
0.9
0.8
0.7
0.6
0.5
8
6
4
2
0
$F(x_2, x_3)$
1
0.8
0.6
0.4
0.2
x_3

But other more complicated inflationary theories (e.g., with non-standard kinetic terms) may produce other bispectra

equilateral



Self-ordering scalar fields



Schmidt-MK: Generalize for other (non-local-model) bispectra

Scale-dependent bias depends only on squeezed limit of bispectrum

No scale-dependent bias for equilateral/SOSF

$b \sim 1/k$ for orthogonal model

Technical innovation: Write

$$\hat{\phi}(\vec{x}) = \phi(\vec{x}) + f_{\rm NL} \int d^3\vec{y}\, d^3\vec{z}\, W(\vec{y}-\vec{x}, \vec{z}-\vec{x})\phi(\vec{y})\phi(\vec{z})$$

$$\hat{\tilde{\phi}}(\vec{k}) = \tilde{\phi}(\vec{k}) + f_{\rm NL} \int \frac{d^3\vec{k}_1}{(2\pi)^3}\, \widetilde{W}(\vec{k}_1, \vec{k}-\vec{k}_1)\tilde{\phi}(\vec{k}_1)\tilde{\phi}(\vec{k}-\vec{k}_1)$$

$$\widetilde{W}(k_1, k_2, k_3) = \frac{1}{2f_{\rm NL}} \frac{B_\phi(k_1, k_2, k_3)}{P_{\phi1}P_{\phi2} + P_{\phi1}P_{\phi3} + P_{\phi2}P_{\phi3}}$$

Odd-Parity CMB Bispectra (with T. Souradeep 2010)

- On full sky

$$T(\hat{n}) \qquad \rightarrow \qquad a_{lm} = \int Y^*_{lm}(\hat{n}) T(\hat{n})$$

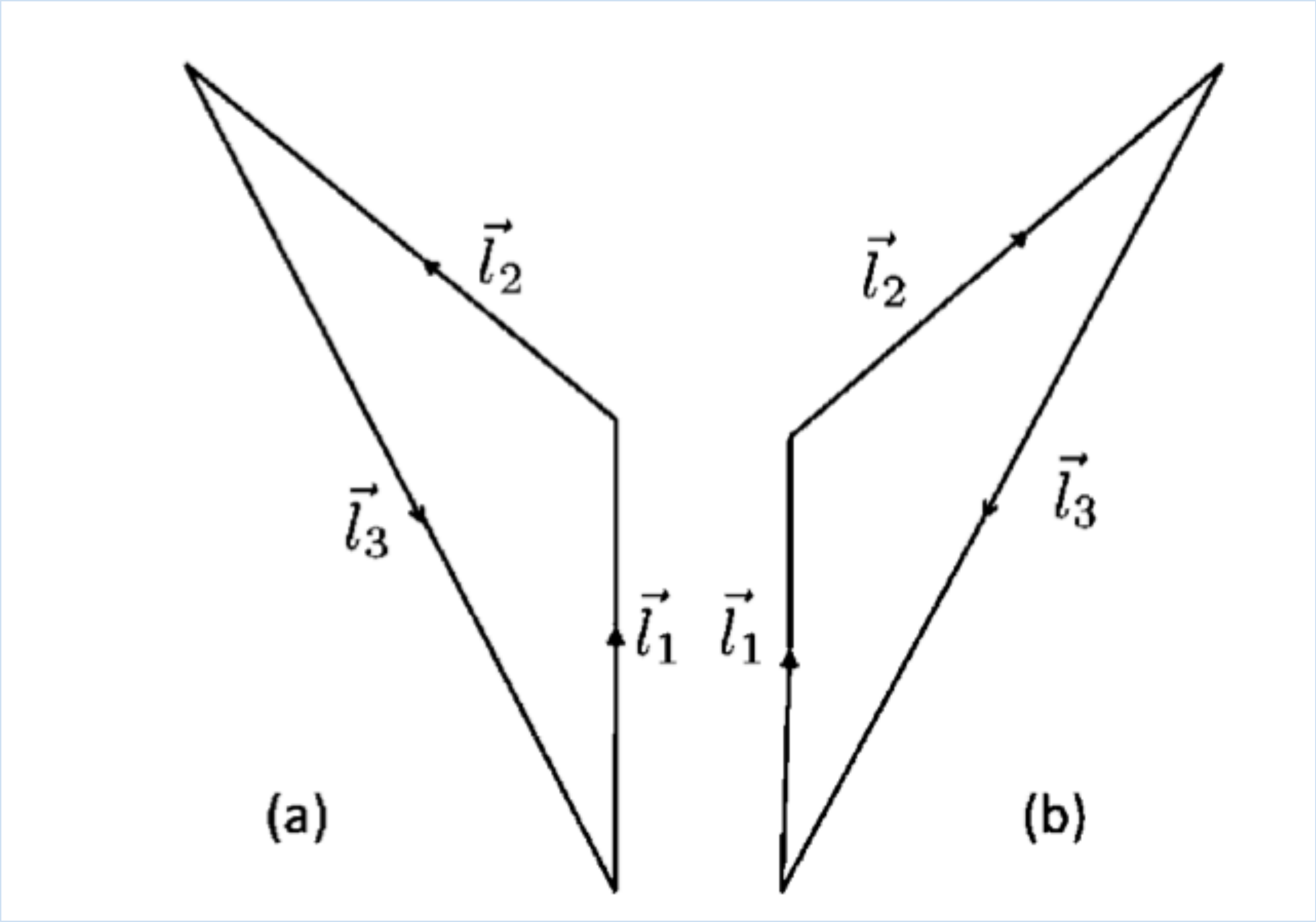
Bispectrum is then

$$B(l_1, l_2, l_3) \propto \sum_{\{lm\}} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}$$

for {lm} that satisfy triangle relations and also $l_1+l_2+l_3$=even.

Restriction $l_1+l_2+l_3$=even assumes CMB does not have a preferred parity.
But can construct bispectrum that probes parity-breaking 3-pt correlations with $l_1+l_2+l_3$=odd.

Statistic probes difference between correlations of triangles with opposite parity



Exotic physics to produce such correlations would have to be *very* exotic.

Measurement can be used as null test (a parity “jackknife” test) or consistency check for complicated analyses.

Statistics of f_{nl} estimators (W. T. Smith)

Suppose CMB experiment measures f_{nl}=30 with standard error σ=10. What does this mean?

If PDF for f_{nl} estimators is Gaussian, then is 3σ departure null hypothesis f_{nl}=0.

Or if measurement is f_{nl}=0 with standard error σ=10 and errors are Gaussian, then is inconsistent at 3σ level with f_{nl}=30.

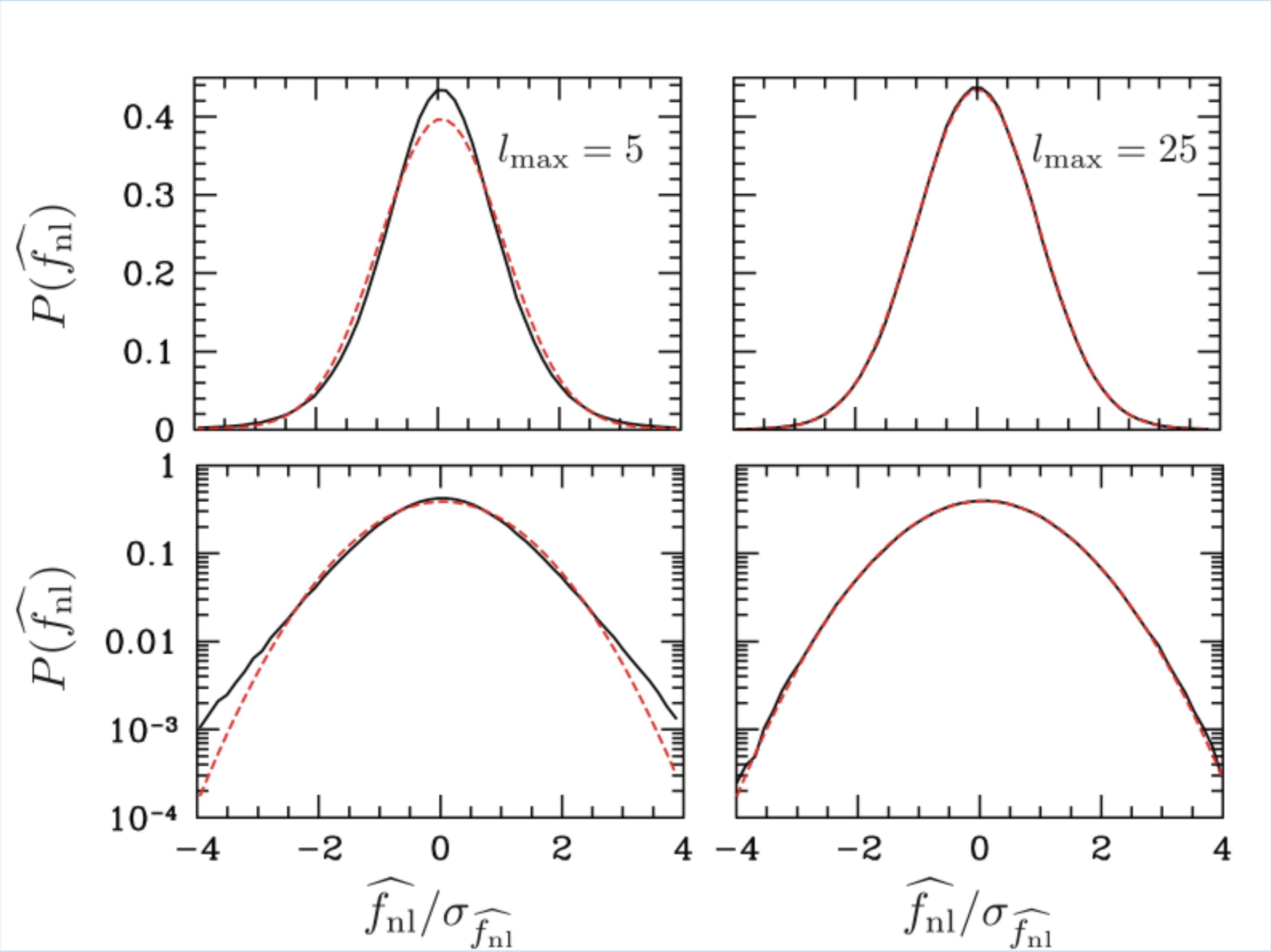
But are the errors Gaussian?

f_{nl} estimator composed of sums of triples of Gaussian random variables:

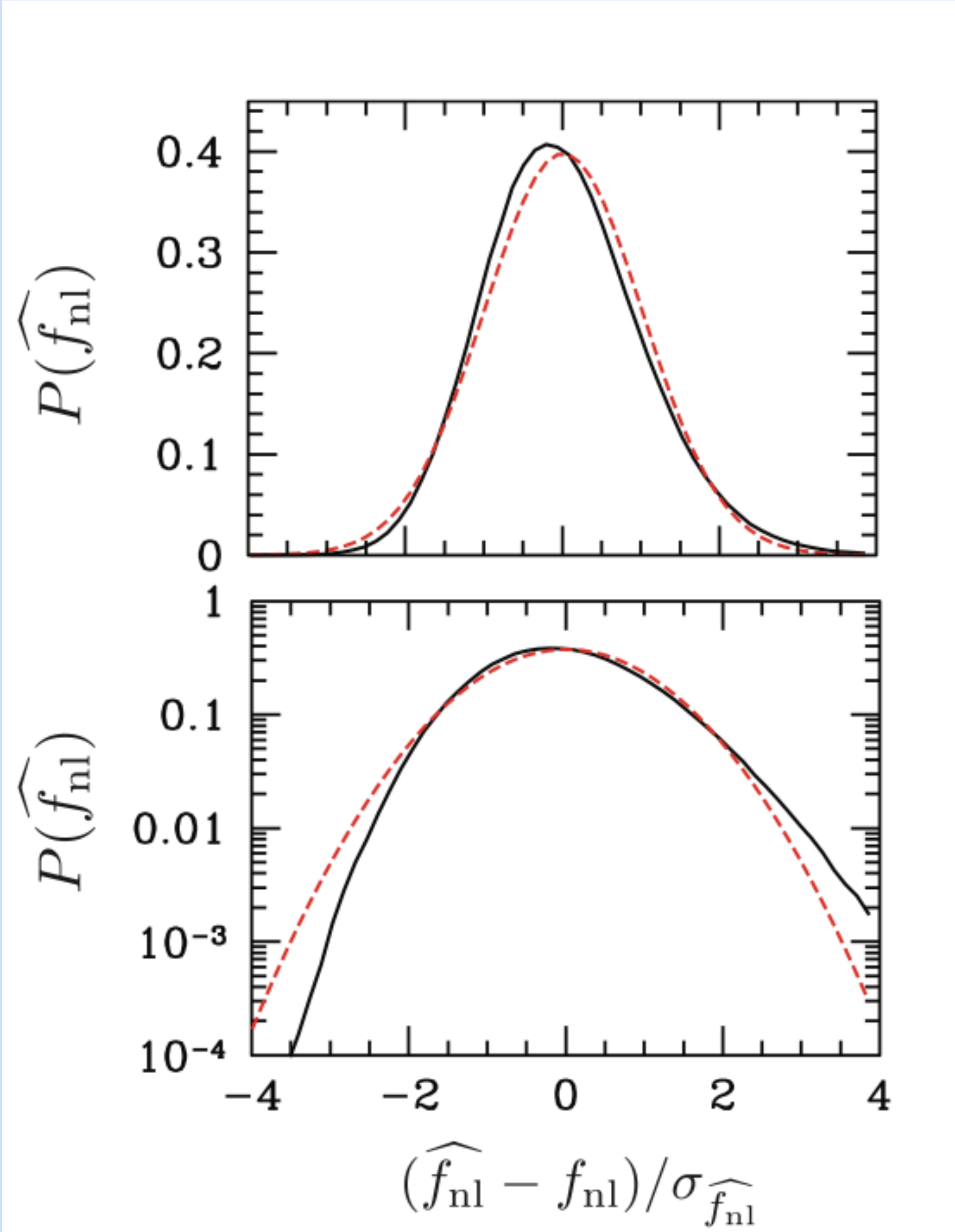
$$\widehat{f_{\rm nl}} \equiv \sigma^2_{f_{\rm nl}} \sum_{\vec{l}_1+\vec{l}_2+\vec{l}_3=0} \frac{T_{\vec{l}_1} T_{\vec{l}_2} T_{\vec{l}_3} B(l_1, l_2, l_3)/f_{\rm nl}}{6\Omega^2 C_{l_1} C_{l_2} C_{l_3}}$$

Contains ~N^2 >> N (number data points) triples, so central-limit theorem does not apply, and PDF for $\widehat{f_{\rm nl}}$ not necessarily Gaussian

Monte Carlo results: PDF *is* ~Gaussian if f_{nl}=0



But not if $f_{nl} \neq 0$.

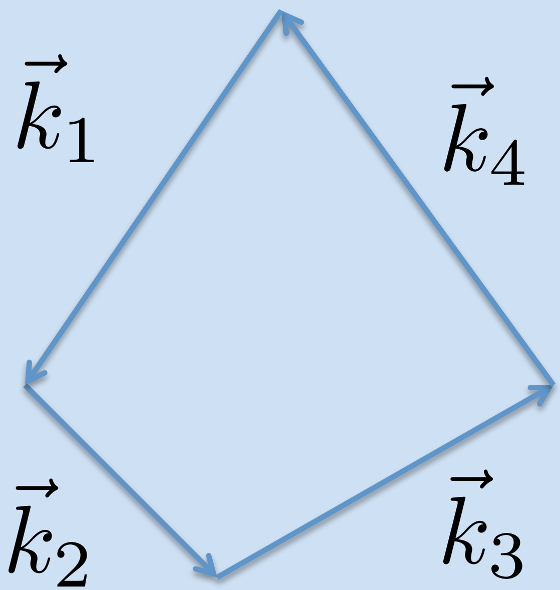


3σ departure from null hypothesis *does* represent 99.7% CL departure

But more care must be taken in ruling out nonzero f_{nl} from null result; and null result actually a bit more constraining than assumption of Gaussian PDF would suggest

Beyond the bispectrum: The trispectrum

$$\left\langle \Phi_{\vec{k}_1} \Phi_{\vec{k}_2} \Phi_{\vec{k}_3} \Phi_{\vec{k}_4} \right\rangle = \mathcal{T}(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) \delta_{\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4, \vec{0}}$$



e.g., MK, Smith, Heavens 2011